科目：微積分 適用：科院三系聯招

編號： 401

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

(以下各題均須寫出計算過程方予計分)

1. **(10%) The region bounded by the curves $y = x^2 - 2x$ and $y = 3x$ is revolved around the line $y = -1$. Find the volume of the solid that is generated.**

2. **(10%) Find the function $f(x)$, such that** $\int_2^{x^3} f(t)dt = \sqrt{3x^3+1} - 5$.

3. **(10%) Sketch the graph of the function** $f(x) = 3x^{\frac{5}{3}} - 5x$.

4. **(10%) Evaluate** $\int \frac{dx}{1+\sqrt{x}}$.

5. **(10%) Find an equation(s) for the tangent(s) to the curve** $x(t) = t^3,\ y(t) = 1-t,\ t \in (-\infty, \infty)$, **at the point (8,-1).**

6. **(10%) Let $f(x) = e^x$**
   - (a) **(5%) Find Taylor series in x of $f(x)$.**
   - (b) **(5%) Show that the series** $\sum \frac{1}{k!} x^k$ **is convergent for all real number x.**

7. **(10%) Set** $g(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$
   - (a) **(5%) Show that** $\frac{\partial g}{\partial x}$ *and* $\frac{\partial g}{\partial y}$ both exist at (0,0). What are their values at (0,0).
   - (b) **(5%) Show that $g(x,y)$ is discontinuous at (0,0).**

8. **(30%) [Hint : if $f'(t) = kf(t)$, then $f(t) = f(0)e^{kt}$]**
   - (a) **(5%) Find the gradient $\nabla f(x,y)$, where $f(x,y) =$** $x^2 + y^2$.
   - (b) **(5%) Find the directional derivative of the function f at the point (1, 2) in the direction of the vector 2i-3j.**
   - (c) **(10%) Determine the path of steepest descent along the surface $z = x^2 + y^2$ from the point (1, 2, 5).**
   - (d) **(5%) Determine the level curve of f that passes through the point (1, 2).**
   - (e) **(5%) Show that the gradient vector $\nabla f(1, 2)$ is perpendicular to the level curve of f that passes through the point (1, 2).**