一、填空題(共 60 分，每空格 5 分，不需列出計算過程)

1. Find the third Taylor polynomial of $f(x) = \ln(x+1)$ at $x=0$.

Ans:_____

2. R is bounded by $y = \sqrt{x}, y = x^2$, $\iint_R (x+y)dA$

=_____

3. Find the slope of the line that is tangent to the curve

$x^2y^3 - 6 = 5y^3 + x$ when $x=2$. Ans:_____

4. Let $f(x) = \dfrac{(x^2-x)e^{x-1}}{x^2+3x+2}$, find $f'(x)$ =_____

5. $\int_{-2}^{2} \dfrac{xe^{x^2}}{x^2+1}dx$ =_____.

6. $f(x) = \dfrac{2}{1+x^2}$ Find the inflection points, if any, of the function.

Ans:_____

7. Find following integral.

(a) $\int_0^1 \dfrac{x^3}{\sqrt{3+x^2}}dx$ =_____.

(b) $\int_0^8 \int_{x/2}^4 \sqrt{y^2+4}\,dydx$ =_____.

8. (a) Evaluate $\lim_{x\to 0} \dfrac{e^x}{2x^3+9x^2-11x}$=_____.

(b) Evaluate $\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=$ _____.

9. (a) Find $\frac{dy}{dx}=$ _____ for $y\ln x+2=x^{3/2}y^{5/2}$.

(b) Find $\frac{dy}{dx}=$ _____ for $y=(x^3+2)(x^2-1)^4$.

二、計算題(共 40 分，每題 10 分，沒有列出計算過程者不予計分)

1. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x,y)=e^{x-y}$ subject to the constraint $x^2+y^2=1$. (10%)

2. solve the given initial value problem $y=f(x)$, $\frac{dy}{dx}=\frac{\ln\sqrt{x}}{x}$ where $y=2$ when $x=1$ (10%)

3. Approximate the amount of material needed to make a water tumbler of diameter 3 cm and height 9 cm. Assume the walls of the tumbler are 0.2 cm thick. (10%)

4. A manufacturing firm estimates that its total production of automobile batteries in thousands of units is

$$f(x,y)=3x^{1/3}y^{2/3}$$

where $x$ is the number of units of labor and $y$ is the number of units of capital utilized. Labor costs are \$80 per unit, and capital costs are \$150 per unit. How many units each of labor and capital will maximize production, if the firm can spend \$40,000 for these costs? (10%)