

科目：微積分 適用：資工系二

編號：312

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題

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(以下各題均須寫出計算過程方予計分)

1. (10%) 以  $\varepsilon$ - $\delta$  證明 (i.e. find the largest  $\delta$  that works for the given  $\varepsilon$ )

$$\lim_{x \rightarrow 4} \sqrt{x} = 2.$$

2. (20%)

(a) (10%) Please describe the mean-value theorem and prove it.

(b) (10%) Set  $f(x) = x^{-1}$ ,  $a = -1$ ,  $b = 1$ . Verify that there is no number  $c$  for which  $f'(c) = [f(b) - f(a)] / (b - a)$ . Explain how this does not violate the mean-value theorem.

3. (10%) Find  $A$  and  $B$  given that the derivative of

$$f(x) = \begin{cases} Ax^2 + B, & x < -1 \\ Bx^3 + Ax + 4, & x \geq -1 \end{cases}$$

is continuous for all  $x$ .

4. (10%) Estimate  $f(5.4)$  given that  $f(5) = 1$  and  $f'(x) = \sqrt[3]{x^2 + 2}$ .

5. (10%) Find the volume of the solid generated by revolving the region between  $y = \sqrt{x}$ ,  $0 \leq x \leq 1$  and  $y = x^2$ ,  $0 \leq x \leq 1$ , around the line  $x = -2$ .

6. (10%) Find the least upper bound and greatest low bound of the sequence

$$\left\{ a_n = \frac{n}{e^n} \right\}$$

7. (10%) Find values for  $a$  and  $b$  so that  $\lim_{x \rightarrow 0} \frac{\sin 2x + ax + bx^3}{x^3} = 0$ .

8. (10%) Let  $r$  be a positive number. For what values of  $r$  (if any) does

$$\sum_{k=0}^{\infty} \frac{r^k}{k^r} \text{ converge?}$$

9. (10%) Find the Taylor series of  $f(x) = 1/\sqrt{1-x^2}$  in powers of  $x$ .