一、填空題(共 55 分，每空格 5 分，不需列出計算過程)

1. $f(x,y)=\int_x^{y^2}\ln(t^2+1)dt$ Evaluate

$f_{xx}(x,y)=$_____,

$f_{yy}(x,y)=$_____,

$f_{xy}(0,0)=$_____.

2. Evaluate

$$\lim_{n\to\infty}\sqrt[n]{\frac{n!}{n^n}}=\text{\_\_\_\_\_}.$$

3. Find the asymptotes of the function.

(a) $f(x)=\dfrac{x^3-x}{x(x+1)}$

Ans:_____

(b) $g(x)=\dfrac{3x}{x^2-x-6}$

Ans:_____

4. Evaluate

$$\lim_{x\to 0}\frac{(5+x)\ln(x+1)}{e^x-1}=\text{\_\_\_\_\_}.$$

5. Find the derivate of following function.

$$f(x)=\frac{\sqrt{2x+1}}{x^2-1}\quad f'(x)=$$

6. Find following indefinite integral.

(a) $\int \frac{x}{(x^2+1)\ln(x^2+1)}dx=$ _____.

(b) $\int \sqrt{x}\ln\sqrt{x}\,dx=$ _____.

7. Find the value of p for which the series is convergent.

$\sum_{n=1}^{\infty}\frac{\ln n}{n^p}$. Ans:_____

二、計算題(共 45 分，沒有列出計算過程者不予計分)

1. Recognize the given limit as a definite integral and then evaluate that integral by the fundamental Theorem of Calculus.(10%)

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{3i}{n}\right)^2\frac{3}{n}$$

2. Find the absolute extrema of the function $f(x) = xe^{-x^2}$ defined on the interval [-1, 1]. (10%)

3. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{4n^2 - 1}$ is convergent or divergent. (5%)

4. Suppose the quantity demanded per week of a certain dress is related to unit price p by the demand equation $p = \sqrt{800 - x}$, where p is in dollars and x is the number of dresses made. To maximize the revenue, how many dresses should be made and sold each week? (10%)

5. Two chemical solutions, one containing N molecules of chemical A and another containing M molecules of chemical B, are mixed together at time $t = 0$. The molecules from the two chemicals combine to form another chemical solution containing y(AB) molecules. The rate at which the AB molecules are formed, dy/dt, is called the reaction rate and is jointly proportional to (N-y) and (M-y). Thus,

$$\frac{dy}{dt} = k(N-y)(M-y)$$

where k is constant. Solve this differential equation with the side condition $y(0)=0$ assuming that $N-y>0$ and $M-y>0$.(10%)