(以下各題均須寫出計算過程方予計分)

1. (10%) Find an equation for the tangent line to the curve $x^3+y^3=1+3xy^2$ at (2,-1).

2. (10%) Use a differential to estimate $\cos 46^\circ$. ($\sqrt{2}$=1.41421,π=3.14159)

3. (10%) Find the volume of the solid generated by revolving the region bounded by $y=x^2$ and $y=2x$ about the x-axis.

4. (10%) *Definition* : The function $\ln(x) = \int_1^x \frac{dt}{t},\ x > 0,$ is called the natural logarithm function. Prove that $\ln(ab) = \ln(a) + \ln(b)$ for $a, b > 0$.

5. (10%) Find the function $f(x)$, such that $\int_2^{x^3} f(t)dt = \sqrt{3x^3+1} - 5$.

6. (10%) Evaluate $\int \frac{dx}{\sqrt{1-\sqrt{x}}}$

7. (10%)The sequence $\{a_n\}$ defined by $a_n = \frac{n}{n+1}$
   (a) (2%) Show that it is increasing.
   (b) (3%) Find the greatest lower bound.
   (c) (5%) Find the least upper bound.

8. (10%) Verify that the improper integral $\int_1^4 \frac{dx}{(x-2)^2}$ diverges.

9. **(10%) Show that the series $\sum \frac{1}{k} x^k$ converges on [-1, 1).**

10. (10%) **Use the chain rule to find the rate of change of** $f(x,y) = \frac{1}{3}(x^3 + y^3)$ **with respect to** t **along the curve** $r(t) = \cos(t)\,\mathbf{i} + \sin(t)\,\mathbf{j}$.