

科目：微積分 適用：應化系二

編號：323

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題
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第 1 頁

1. A right triangle is formed by the coordinate axes and a line through the point (2, 5). Express the area of the triangle as a function of the x -intercept and calculate the minimal area of the triangle. (20%)

2. Evaluate the following integrals:

(a) $\int e^{2x} \sin 2x \, dx$ (10%) (b) $\int \frac{3x^2}{(x^3+2)^2} \, dx$ (10%)

(c) $\int \frac{dx}{\sqrt[3]{x} + \sqrt{x}}$ (10%) (d) $\int_0^1 \int_{\sqrt{x}}^1 \sin\left(\frac{y^3+1}{2}\right) \, dy \, dx$ (10%)

3. When an object of mass m moves through a viscous medium, it is acted on by a frictional force that acts in the direction opposite to its motion.

This frictional force depends on the velocity of the object and is given by

$$F(v) = -\alpha v - \beta v^2, \text{ where } \alpha \text{ and } \beta \text{ are positive constants. (a) From the}$$

Newton's second law, $F = m a$, solve the differential equation to find

$$v = v(t). \text{ (10\%)} \quad \text{(b) What happens to } v(t) \text{ as } t \rightarrow \infty. \text{ (10\%)}$$

4. Show that the f function ($f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$) satisfies Laplace's equation in three dimensions. (20%)