

科目：工程數學 適用：土木系三

編號：811

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

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1. (32%, each 8%) Solve the following ordinary differential equations.

(a) $2xyy' = y^2 - x^2, y(4) = 0.$

(b) $2 \sin y^2 dx + xy \cos y^2 dy = 0, y(2) = \sqrt{\pi/2}.$

(c) $4y'' - 4y' - 3y = 0, y(-2) = e, y'(-2) = -e/2.$

(d) $x^2 y'' - 3xy' + 3y = 3 \ln x - 4, y(1) = 0, y'(1) = 1.$

2. (20%, each 10%) The gamma function is defined as

$$\Gamma(\nu) = \int_0^{\infty} e^{-t} t^{\nu-1} dt, \nu > 0.$$

(a) Show that $\Gamma(\nu+1) = \nu \Gamma(\nu)$, for arbitrary $\nu > 0$.

(b) Show that $\Gamma(n+1) = n!$, if n is an integer.

3. (12%) The Laplace transform of a function
- $f(t)$
- is defined as

$$F(s) = \int_0^{\infty} e^{-st} f(t) dt.$$

Please find $f(t)$ if $F(s) = \frac{1}{s^3 + s^5}.$

4. (12%) Find the Fourier series of the function
- $\cos^2 t + \sin^3 t + \cos^4 t.$

5. (12%) Evaluate the surface integral
- $\iint_S (3xi + 4yj - 5zk) \cdot n dA$
- over the

sphere $S: x^2 + y^2 + z^2 = \pi.$

6. (12%) Evaluate the line integral
- $\int_C \mathbf{F} \cdot d\mathbf{r}$
- where
- $\mathbf{F} = [y, \frac{1}{2}z, \frac{3}{2}y]$
- and

C is the circle $x^2 + y^2 + z^2 - 6z = 0$ and $z = x + 3.$