科目：微積分　適用：資工系二

編號：312

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

本試題
共 1 頁
第 1 頁

1. (20%)

   (a) (10%) Please give an ε and δ and show that $\lim_{x \to c} |x| = |c|$.

   　(Hint: $||a|-|b|| \leq |a-b|$)

   (b) (10%) Let g be a function defined at least on some open interval containing the number 0. We say that g is of *smaller order than h*, or that $g(h)$ is *little-o(h)* and write $g(h) = o(h)$, if and only if $g(h)$ is small enough compared with h that $\lim_{h \to 0} \dfrac{g(h)}{h} = 0$. Show that $h^3 = o(h)$.

2. (15%) Show that $\ln(1+x) \approx x$ as $x \to 0$. (Hint: $(\ln(x))' = 1/x$)

3. (20%) Let F be defined by $F(x) = \int_0^x \dfrac{1}{1+t^2} dt$, where x is any real number. (Hint: $F'(x) = \dfrac{1}{1+x^2}$)

   (a) (10%) Find the critical numbers of F and determine the intervals on which F is increasing and the intervals on which F is decreasing.

   (b) (10%) Determine the concavity of the graph of F and find the points of inflection (if any).

4. (20%)

   (a) (10%) Verify that the region below the graph of $f(x) = 1/x$, $x \geq 1$ has infinite area.

   (b) (10%) Suppose that the region (a) is revolved about the x-axis, what is the volume of the resulting solid ?

5. (25%) The distribution of voltage on a metal plate is given by $V(x,y) = 50-x^2-4y^2$.

   (a) (5%) At the point (1,-2), in what direction does the voltage increase most rapidly ?

   (b) (5%) In what direction does the voltage decrease most rapidly ?

   (c) (5%) What is the magnitude of this increase or decrease ?

   (d) (10%) Describe the path of a particle that starts at the point (1,-2) and moves in the direction of greatest voltage increase.