

科目：統計學 適用：經濟系三

編號：723

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

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壹. Mr. White proposed a method (White's test) to test whether a coin is unbiased as follows. Toss this coin 5 times independently, and conclude it is biased only if the head appear great or equal to 4 times. Please answer the following questions.

- (1) (10%) What is the type I error of White's test (assume the coin is unbiased)?
- (2) (10%) If the head appear with the probability 0.6 (the coin is indeed biased), what is the power of White's test?
- (3) (10%) How will you test whether this coin is unbiased? (Please explain carefully about the null hypothesis, significance level, rejection region and the test statistic.)

貳. The joint density function of  $X$  and  $Y$  is

$$f_{X,Y}(a,b) = \begin{cases} a + kb & \text{if } 0 < a < 1, 0 < b < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (1). (10%) What is the value of  $k$  such that  $f_{X,Y}(a,b)$  is a probability density function?
- (2). (10%) Find the density function of  $X$ .
- (3). (10%) Find  $P\{X + Y < 1\}$ .

參. Let the model of regression analysis be  $Y_i = \alpha_0 + \beta_0 X_i + U_i$ . Assume that  $X_i$ 's are fixed numbers and  $U_i$  satisfied the following properties:

(i)  $E(U_i) = 0, i=1, \dots, n.$

(ii)  $\text{var}(U_i) = \sigma_0^2, i=1, \dots, n;$  and  $\text{cov}(U_i, U_j) = 0$  for  $i \neq j.$

- (1) (20%) Please show that the ordinary least squares estimators are

$$\hat{\beta}_n = \frac{\sum_{i=1}^n (X_i - \bar{X}_n)(Y_i - \bar{Y}_n)}{\sum_{i=1}^n (X_i - \bar{X}_n)^2} \quad \text{and} \quad \hat{\alpha}_n = \bar{Y}_n - \hat{\beta}_n \bar{X}_n$$

- (2) (20%) Please show that  $\hat{\alpha}_n$  and  $\hat{\beta}_n$  are linear unbiased estimators of  $\alpha_0$  and  $\beta_0$ .