

科目：微積分 適用：資工系二

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. (15%) Sketch the graph of the function $f(x) = \ln\left(\frac{x^4}{x-1}\right)$.

2. (15%) Find $\lim_{n \rightarrow \infty} \frac{1}{n} \left\{ \frac{1}{\sqrt{1^2 + n^2}} + \frac{2}{\sqrt{2^2 + n^2}} + \cdots + \frac{n}{\sqrt{n^2 + n^2}} \right\}$.

[Hint: limit of Riemann sums]

3. (10%) Calculate $\int e^x \cos x \, dx$ Hint: $\cos x \, dx = d \sin x$ & integration by parts

4. (10%) Find the area enclosed by the ellipse $(x^2/a^2) + (y^2/b^2) = 1$.
(Hint: $\cos^2 x = (1 + \cos 2x)/2$.)

5. (10%) Let
$$g(x, y) = \begin{cases} \frac{x^2 y^2}{x^4 + y^4}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Show that $\lim_{(x, y) \rightarrow (0, 0)} g(x, y)$ does not exist.

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6. (20%) Let $f(x,y) = 3x^2 + y$.(a) (5%) Find $\lim_{h \rightarrow 0} \frac{f(x(2+h), y(2+h)) - f(2,4)}{h}$ giventhat $x(t)=t$ and $y(t)=t^2$. (These functions parametrize the parabola $y=x^2$)(b) (5%) Find $\lim_{h \rightarrow 0} \frac{f(x(4+h), y(4+h)) - f(2,4)}{h}$ giventhat $x(t)=1/4(t+4)$ and $y(t)=t$. (These functions parametrize the line $y=4x-4$)(c) (5%) Compute the directional derivative of f at $(2,4)$ in the direction of $\mathbf{i}+4\mathbf{j}$.(d) (5%) Notice that $\mathbf{i}+4\mathbf{j}$ is a direction vector for the line $y=4x-4$ and this line is tangent to the parabola $y=x^2$ at $(2,4)$. Explain why the computations in (a), (b), and (c) yield different values.7. (10%) Show that the series $\sum_{k=1}^{\infty} \frac{1}{k} x^k$ converges on $[-1, 1)$.8. (10%) Maximize $f(x,y,z) = xyz$ subject to the side condition

$$x^3 + y^3 + z^3 = 1, \text{ with } x \geq 0, y \geq 0, z \geq 0.$$

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