科目：微積分 適用：資管系二 經濟系二 財金系二

編號： 231 222 242

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

一、 填空題(共 60 分，每空格 6 分，不需列出計算過程)

1. Find the derivatives (微分函數) of the following functions:

(1) $f(x) = 3x^2 + 2x + 2$, $f'(x) =$ _____. (6%)

(2) $g(t) = t(3x^2 + 2x + 2)^{-2}$, $g'(t) =$ _____. (6%)

(3) $h(x) = (2x + 1)^4$, $h'(x) =$ _____. (6%)

(4) $m(x) = (3x^2 + 2x + 2)^{-2}(2x + 1)^4$,

$m'(x) =$ _____. (不用展開) (6%)

(5) $\sqrt{xy} = 2x + y^2$, $\frac{dy}{dx} =$ _____. (6%)

2. Evaluate $\iint_R (x^3 + 4y)dA$ =_____, where R is bounded by $y = 2x, y = x^2$ (6%)

3. Find the particular solution of the differential equation $ye^x dx + (y^2 - 1)dy = 0$ that satisfies the condition $y(0) = 1$.

Ans:_____ (6%)

4. Evaluate $\lim_{x \to \infty} \frac{x^3}{e^{2x}}$ =_____. (6%)

5. Evaluate $\lim_{x \to \infty}(1 + \frac{1}{x})^x$ =_____. (6%)

6. Evaluate $\int \frac{e^{\frac{2}{x}}}{x^2} dx$ = _____. (6%)

二、計算題(共 40 分，沒有列出計算過程者不予計分)

1. Let f and g be continuous functions such that $f(x) \geq g(x)$ on the interval $[a, b]$. Then, the area of the region bounded above by $y = f(x)$ and below by $y = g(x)$ on $[a, b]$ is given by

$$\int_a^b [f(x) - g(x)]\, dx$$

Also, given any three noncollinear (不共線) points (e.g., $(x_0, f(x_0))$, $(x_1, f(x_1))$, and $(x_2, f(x_2))$), there is a unique parabola (拋物線) that passes through the given points of the partition. Please show that the area bounded by the parabola and the x axis between $x_1 = x_0$ and $x = x_2$ is given by

$$\frac{\Delta x}{3}[f(x_0) + 4f(x_1) + f(x_2)]$$

square units. (Hint: Suppose that $x_1 - x_0 = x_2 - x_1$) (7%)

2. Show that Simpson's rule:

$$\int_a^b f(x)\, dx \approx \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \ldots + 4f(x_{n-1}) + f(x_n)]$$

where $\Delta x = \frac{b-a}{n}$ (7%)

3. Find approximations (趨近值) of $\int_1^2 \frac{2}{x}\,dx$

(1) by using the integration rule (an example of integration rules: $\int x\,dx = \frac{1}{2}x^2 + c$), and (2%)

(2) by using the Simpson's rule ($n = 6$). (4%)

4. From 3, if the error $\leq 0.0004267 = \frac{4}{15}(25)^{-2}$, what is the least n?

Hints: Let the definite integral be

$$\int_a^b f(x)\,dx$$

and [a, b] is divided by n subintervals.

The maximum error is **EITHER**

$$\frac{M(b-a)^3}{12n^2}$$

where M is a number such that $|f''(x)| \leq M$ for all x in $[a, b]$, **OR**

$$\frac{M(b-a)^5}{180n^4}$$

where M is a number such that $|f^{(4)}(x)| \leq M$ for all x in $[a, b]$. (6%)

5. Find the area of the region bounded by the graphs of $x = y^2$ and $y = x - 2$. (7%)

6. Express the number $3.2\overline{14} = 3.2141414\ldots$ as a rational number. (7%)