

科目：工程數學 適用：土木系三

編號：811

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

 本試題  
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 第壹頁

1. (18%) Solve the differential equation  $y'' + \omega^2 y = p_0 \sin \omega t$ , where  $y = y(t)$ . The initial conditions are  $y(0) = 0$ ,  $y'(0) = 0$ .

2. (10%) Write the general solution of the differential equation in terms of Bessel functions  $J_n(x)$  or  $Y_n(x)$ :  $x^2 y'' + xy' + (x^2 - 36)y = 0$

3. (18%) Calculate the value of the determinant:

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 6 & 7 & 3 & 0 \\ 8 & 9 & 0 & 4 \end{vmatrix}$$

4. (18%) Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix}$$

5. (18%) Solve the linear system of equations:

$$2x_1 - 3x_2 + x_3 + 2x_4 = -1$$

$$x_1 + x_3 - x_4 = 3$$

$$-2x_2 - 2x_3 + 3x_4 = -5$$

$$4x_1 + 2x_2 + 3x_3 - x_4 = 11$$

6. (18%)  $\vec{F} = 7x\vec{i} - z\vec{k}$ , calculate the surface integral  $\iint_S \vec{F} \cdot \vec{n} dA$  over a

sphere  $S: x^2 + y^2 + z^2 = 4$ , where  $dA$  is an infinitesimal area on the sphere and  $\vec{n}$  is the unit normal vector at  $dA$ .