

科目：離散數學 適用：資工系三

編號：822

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題
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第 1 頁

一 簡答題 (20%，每小題 2%)

1. If $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$, and $C = \{4, 5, 6\}$, determine the following:
- (a) $A \times (B \cap C)$.
 - (b) The number of relations from B to C .
 - (c) The number of relations from A to B that contain exactly three ordered pairs.
 - (d) How many functions are one-to-one from A to B ?
 - (e) How many functions are onto from B to A ?
 - (f) How many closed binary operations are there on B ?
 - (g) How many functions $f: B \rightarrow A$ satisfy $f_{\{2,3\}}(\{2,3\}) = \{1,2\}$?
 - (h) How many closed binary operations f on B satisfy $f(1,2) = 5$?
 - (i) How many of the functions f in part (h) have an identity?
 - (j) How many of the functions f in part (h) are commutative?

二 計算與證明題 (80%，每題 10%) (以下各題均須寫出計算或證明過程方予計分)

1. In how many ways can 17 be written as a sum of 2's and 3's if the order of the summands is
 - (a) not relevant? (5%)
 - (b) relevant? (5%)
2. At a high school science fair, 34 students received awards for scientific projects. Fourteen awards were given for projects in biology, 13 in chemistry, and 21 in physics. If three students received awards in all three subject areas, how many received awards for exactly
 - (a) one subject area? (5%)
 - (b) two subject areas? (5%)
3. (10%) If $n \in \mathbb{Z}^+$, how many possible values are there for $\gcd(n, n + 3000)$?
4. (10%) For all $n \in \mathbb{Z}^+$ prove that if $n \geq 24$, then n can be written as a sum of 5's and/or 7's.
5. (10%) If a, b are relatively prime and $a \geq b$, prove that $\gcd(a - b, a + b) = 1$ or 2 .
6. (10%) For $n, k \in \mathbb{Z}^+$, prove that $\lceil n/k \rceil = \lfloor (n-1)/k \rfloor + 1$.
7. (10%) An auditorium has a seating capacity of 800. How many seats must be occupied to guarantee that at least two people seated in the auditorium have the same first and last initials?
8. (10%) If $\{x_1, x_2, \dots, x_7\} \subseteq \mathbb{Z}^+$, show that for some $i \neq j$, either $x_i + x_j$ or $x_i - x_j$ is divisible by 10.