

科目：微積分 適用：資管系二 經濟系二 財金系二

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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一、填充題(共 60 分，每空格 6 分，不需列出計算過程)

1. Find the derivatives (微分函數) of the following functions:

(1)  $f(x) = 3x^2 + 2x + 2$ ,  $f'(x) =$  \_\_\_\_\_ (6%)

(2)  $g(t) = t(3x^2 + 2x + 2)^{-2}$ ,  $g'(t) =$  \_\_\_\_\_ (6%)

(3)  $h(x) = (2x + 1)^4$ ,  $h'(x) =$  \_\_\_\_\_ (6%)

(4)  $m(x) = (3x^2 + 2x + 2)^{-2}(2x + 1)^4$ ,  
 $m'(x) =$  \_\_\_\_\_ (不用展開) (6%)

(5)  $\sqrt{xy} = 2x + y^2$ ,  $\frac{dy}{dx} =$  \_\_\_\_\_ (6%)

2. Evaluate  $\iint_R (x^3 + 4y) dA =$  \_\_\_\_\_, where R is bounded by  
 $y = 2x, y = x^2$  (6%)

3. Find the particular solution of the differential equation

$ye^x dx + (y^2 - 1)dy = 0$  that satisfies the condition  $y(0) = 1$ .

Ans: \_\_\_\_\_ (6%)

4. Evaluate  $\lim_{x \rightarrow \infty} \frac{x^3}{e^{2x}} =$  \_\_\_\_\_ (6%)

5. Evaluate  $\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x =$  \_\_\_\_\_ (6%)

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6. Evaluate  $\int \frac{e^{\frac{2}{x}}}{x^2} dx =$  \_\_\_\_\_ . (6%)

二、計算題(共 40 分，沒有列出計算過程者不予計分)

1. Let  $f$  and  $g$  be continuous functions such that  $f(x) \geq g(x)$  on the interval  $[a, b]$ . Then, the area of the region bounded above by  $y = f(x)$  and below by  $y = g(x)$  on  $[a, b]$  is given by

$$\int_a^b [f(x) - g(x)] dx$$

Also, given any three noncollinear (不共線) points (e.g.,  $(x_0, f(x_0))$ ,  $(x_1, f(x_1))$ , and  $(x_2, f(x_2))$ ), there is a unique parabola (拋物線) that passes through the given points of the partition. Please show that the area bounded by the parabola and the  $x$  axis between  $x = x_0$  and  $x = x_2$  is given by

$$\frac{\Delta x}{3} [f(x_0) + 4f(x_1) + f(x_2)]$$

square units. (Hint: Suppose that  $x_1 - x_0 = x_2 - x_1$ ) (7%)

2. Show that Simpson's rule:

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4) + \dots + 4f(x_{n-1}) + f(x_n)]$$

where  $\Delta x = \frac{b-a}{n}$  (7%)

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3. Find approximations (趨近值) of  $\int_1^2 \frac{2}{x} dx$

(1) by using the integration rule (an example of integration rules:

$$\int x dx = \frac{1}{2}x^2 + c), \text{ and } (2\%)$$

(2) by using the Simpson's rule ( $n=6$ ). (4%)

4. From 3, if the error  $\leq 0.0004267 = \frac{4}{15} (25)^{-2}$ , what is the least  $n$ ?

Hints: Let the definite integral be

$$\int_a^b f(x) dx$$

and  $[a, b]$  is divided by  $n$  subintervals.

The maximum error is **EITHER**

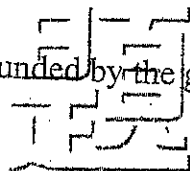
$$\frac{M(b-a)^3}{12n^2}$$

where  $M$  is a number such that  $|f''(x)| \leq M$  for all  $x$  in  $[a, b]$ , OR

$$\frac{M(b-a)^5}{180n^4}$$

where  $M$  is a number such that  $|f^{(4)}(x)| \leq M$  for all  $x$  in  $[a, b]$ . (6%)

5. Find the area of the region bounded by the graphs of  $x = y^2$  and  $y = x - 2$ . (7%)



6. Express the number  $3.\overline{214} = 3.2141414 \dots$  as a rational number. (7%)