科目：統計學　適用：經濟系三

編號：723

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

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(1) Let Y be a random variable with $p(y)$ given in the following table.

Find $E(Y)$, $E(1/Y)$, $E(Y^2-1)$, and $Var(Y)$. (5 points each)

y	1	2	3	4
$p(y)$	0.4	0.3	0.2	0.1

(2) An oil exploration firm is formed with enough capital to finance 10 explorations. The probability of a particular exploration being successful is 0.1. Assume the explorations are independent. Answer the following questions.

(a) Find the mean and variance of the number of successful explorations.

(b) Suppose the firm has a fixed cost of $20000 in preparing equipment prior to doing its first exploration. If each successful exploration costs $30000 and each unsuccessful exploration costs $15000, find the expected total cost to the firm for its 10 explorations. (10 points each)

(3) Let X be a uniform random variable with mean 0, and variance 1. Find the cumulative distribution function (cdf) of X. (10 points)

(4) Let Y_1 and Y_2 be random variables with the following joint probability density function (pdf):

$$f(y_1, y_2) = \begin{cases} 4y_1 y_2, & 0 \le y_1 \le 1, 0 \le y_2 \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Are Y_1 and Y_2 independent? (10 points)

(5) Suppose that $E(\hat{\theta}_1) = E(\hat{\theta}_2) = \theta$, $Var(\hat{\theta}_1) = \sigma_1^2$, $Var(\hat{\theta}_2) = \sigma_2^2$. Consider the estimator $\hat{\theta}_3 = a\hat{\theta}_1 + (1-a)\hat{\theta}_2$.

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(a) Show that $\hat{\theta}_3$ is an unbiased estimator for θ.

(b) If $\hat{\theta}_1$ and $\hat{\theta}_2$ are independent, how should the constant a be chosen to minimize the variance of $\hat{\theta}_3$. (10 points each)

(6) In testing the hypotheses $H_0: \mu = 15$ vs. $H_1: \mu \neq 15$, the following information are given: $\sigma = 5$, $n = 16$, $\bar{x} = 17.4$, $\alpha = 0.05$.

(a) Calculate the value of the test.

(b) Determine the p-value. (10 points each)