

科目：工程數學 適用：電機系三

編號：832

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. (10%) True or false.

- (a) (2%) A diagonalizable matrix is always invertible.
- (b) (2%) An invertible matrix is always diagonalizable.
- (c) (2%) Two similar matrices share the same eigenvalues.
- (d) (2%) Orthogonal vectors are always linearly independent.
- (e) (2%) A is an m -by- n matrix. If $m = n$, then the row space of A equals the column space.

2. (10%) Given $A = \begin{bmatrix} 0.2 & 0.8 \\ 0.8 & 0.2 \end{bmatrix}$, find $\lim_{n \rightarrow \infty} A^n$.3. (10%) Let W be the plane spanned by $\mathbf{a} = (1, 1, 0)$ and $\mathbf{b} = (-1, 0, -1)$.

- (a) (5%) Find orthonormal vectors \mathbf{q}_1 and \mathbf{q}_2 in W by Gram-Schmidt process.
- (b) (5%) Which vector in this plane W is closest to $(1, 0, 0)$?

4. (10%) Given $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix}$, find the orthogonal matrix Q and the diagonal matrix Λ to diagonalize the matrix A so that $A = Q\Lambda Q^T$.5. (10%) Given $A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$. Show that A is invertible if $a \neq 0$ and $a \neq b$.

6. (15%) Solve the following initial value problems:

- (a) (5%) $y'' + 2y' - 3y = 10\delta(x-2) + 10\delta(x-3)$, $y(0) = 1$ and $y'(0) = 0$.
- (b) (5%) $y' + 0.5y = 17\sin(2x)$, $y(0) = -1$.
- (c) (5%) $2xyy' = 3y^2 + x^2$, $y(1) = 2$.

7. (15%) Find a general solution for each of the following ordinary differential equations (ODEs):

- (a) (5%) $y''' + y = 0$.
- (b) (5%) $y'' + 2y' + y = 2x\sin(x)$.
- (c) (5%) $4x^2y''' + 12xy' + 3y = 0$.

8. (10%) Find the Laplace transforms of the following functions:

- (a) (5%) $\sin^2(5t) + t\cos(5t)$.
- (b) (5%) $\cosh^2(3t)$.

9. (10%) Find the inverse Laplace transforms of the following functions:

- (a) (5%) $\frac{1}{s^2 + s/3}$.
- (b) (5%) $\frac{1}{s^2 + 4} - \frac{1}{s + 4}$.