科目：微積分 適用：資工系二

編號： 312

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

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(以下各題均須寫出計算過程方予計分)

1. (15%)
   (a) (5%) Let $f(x)=x^2$, prove that $f'(x)=2x$ by the definition
   $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
   (b) (5%) Let g be a function of several variables which is defined in some neighborhood of $\mathbf{0}$. We will say that $g(\mathbf{h})$ is $o(\mathbf{h})$ iff
   $$\lim_{\mathbf{h}\to\mathbf{0}}\frac{g(\mathbf{h})}{||\mathbf{h}||}=0.$$
   Prove that $g(\boldsymbol{h})=||\boldsymbol{h}||$ is $o(\boldsymbol{h})$.
   (c) (5%) Let $g(x,y)=x^2+y^2$, prove that $\nabla g(x,y)=(2x,2y)$ by the definition $g(\mathbf{x}+\mathbf{h})-g(\mathbf{x})\equiv\nabla g(\mathbf{x})\cdot\mathbf{h}+o(\mathbf{h})$.
2. (15%) Suppose that the temperature at each point of a metal plane is given by the function $T(x,y)=1+x^2-y^2$. Find the path followed by a heat-seeking particle that originates at (-2, 1).
3. (10%) Find the directional derivative of the function $g(x,y)=x^2+y^2$ at the point (1, 2) in the direction of the vector $2\mathbf{i}-3\mathbf{j}$.
4. (10%) Use polar coordinates to calculate the volume of a sphere of radis R.
5. (10%) Find the interval of convergence of $\sum\frac{(-1)^k}{k^2 3^k}(x+2)^k$.
6. (10%) Sketch the graph of the function $f(x)=3x^{\frac{5}{3}}-5x$.
7. (10%) Evaluate $\int\frac{dx}{\sqrt{1-\sqrt{x}}}$.
8. (10%) Estimate $f(5.4)$ given that $f(5)=1$ and $f'(x)=\sqrt[3]{x^2+2}$.
9. (10%) The region bounded by the curves $y=x^2-2x$ and $y=3x$ is revolved around the line $y=-1$. Find the volume of the solid that is generated.