

考生注意：1. 依次序作答，只要標明題號，不必抄題。

2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。

3. 試題隨卷繳回。(餘請詳閱試場規則)

The following problems may be answered in Chinese or English. You need to give all details in order to receive any credit (point).

1. Let g be a function defined at least on some open interval containing the number 0. We say that g is of *smaller order than* h , or that $g(h)$ is *little- $o(h)$* and write $g(h) = o(h)$, iff $g(h)$ is small enough compared with h that

$$\lim_{h \rightarrow 0} \frac{g(h)}{h} = 0, \text{ or equivalently, } \lim_{h \rightarrow 0} \frac{g(h)}{|h|} = 0.$$

Figure 1 shows that the graph of a differentiable function f and a line with slope m that passes through the point $(x, f(x))$. The vertical separation at $x+h$ between the line with slope m and the graph of f has been labeled $g(h)$.

- (a) (7%) Calculate $g(h)$.

- (b) (13%) Show that, of all lines that pass through $(x, f(x))$, the tangent line is the line that best approximates the graph of f near the point $(x, f(x))$ by showing that

$$g(h) = o(h) \text{ iff } m = f'(x).$$

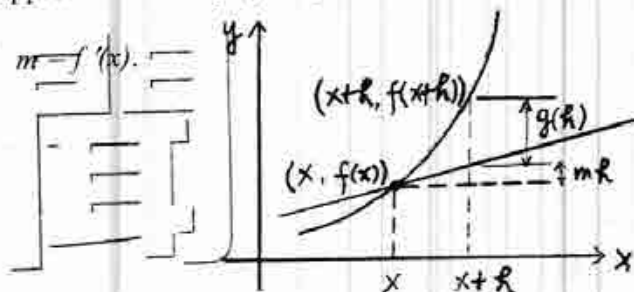


Figure 1

2. Let $f(x, y) = 1 + x^2 + y^2$.

- (a) (6%) Find the points (x, y) , if any, at which $\nabla f(x, y) = 0$.

- (b) (6%) Sketch the graph of the surface $z = f(x, y)$.

- (c) (8%) Determine the path of steepest descent along the surface $z = f(x, y)$ from the point $(1, 1, 3)$.

3. **Definition**: The function $\ln(x) = \int_1^x \frac{dt}{t}$, $x > 0$, is called the natural logarithm function.

- (a) (12%) Prove that $\ln(ab) = \ln(a) + \ln(b)$ for $a, b > 0$.

- (b) (8%) By (a), to prove $\ln(1/b) = -\ln(b)$ and $\ln(a/b) = \ln(a) - \ln(b)$.

4. Let $f(x) = \frac{\ln x}{x}$ on $[1, 2e]$.

- (a) (8%) Find the area of the region R bounded by the graph of f and the x -axis.

- (b) (12%) Find the volume of the solid generated by revolving R around the x -axis.

5. (20%) Verify that the series $\sum_{k=1}^{\infty} \frac{1}{k^2} x^k$ has interval of convergence $[-1, 1]$.