

國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 2 節工程數學 適用：(電機所電子組 425)

(本試題共 1 頁，第 1 頁)

考生注意：1. 依次序作答，只要標明題號，不必抄題。

2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。

3. 試題隨卷繳回。(餘詳閱試場規則)

1. (20 pts.) **What** are the similarities and differences between Fourier transform and Laplace transform? **Take examples** to show when and how to use them.

2. (20 pts.) **Determine** $y(t)$ by solving the following differential equations:

(a) $\frac{dy}{dt} = \cos t + \int_0^t y(\tau) \cos(t-\tau) d\tau$, and $y(0) = 1$.

(b) $\begin{cases} x' = y + \sin t \\ y' = x + 2 \cos t \end{cases}$, and $x(0) = 2, y(0) = 0$.

3. (20 pts.) Consider the triangular pulse signal $x(t)$ as follows:

$$x(t) = \begin{cases} 1 - |t| & |t| \leq 1 \\ 0 & |t| > 1 \end{cases}$$

- (a) **Find** the Fourier transform $X(j\omega)$ of $x(t)$.

- (b) If $\tilde{X}(j\omega) = e^{-j\omega/2} X(j\omega) = e^{-j\omega/2} X(-j\omega)$, **determine** and **sketch** the inverse Fourier transform of $\tilde{X}(j\omega)$.

4. (20 pts.) The current $i(t)$ in an RC series circuit (the resistance $R = 10 \Omega$ and the capacitance $C = 0.5 F$) with the voltage source $e(t)$ can be determined from the integral equation

$$Ri(t) + \frac{1}{C} \int_0^t i(\tau) d\tau = e(t).$$

- (a) **Find** the current $i(t)$ when $e(t) = 5(t^2 + t) V$.

- (b) **Determine** the complex impedance $Z(j\omega) = \frac{E(j\omega)}{I(j\omega)}$, where $E(j\omega)$ and $I(j\omega)$ are the Fourier transform of $e(t)$ and $i(t)$, when $e(t) = \sin 2t V$.

5. (20 pts.) The following differential equation is known as Ricatti's equation:

$$\frac{dy}{dt} = P(t) + Q(t)y + R(t)y^2.$$

- (a) A Ricatti equation can be solved by a succession of two substitutions if we know a particular solution y_p of the equation. First **use** the substitution $y = y_p + x$, and then **discuss** what to do to solve this equation next.

- (b) **Determine** the unique solution for the following differential equation with $y(1) = 0$ and the given particular solution $y_p = \frac{2}{t}$:

$$\frac{dy}{dt} = -\frac{4}{t^2} - \frac{1}{t}y + y^2.$$