

科目：工程數學 適用：土木所結構組 土木所大地組 土木所水利組 考生注意：

地震所

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1. 依次序作答，只要標明題號，不必抄題。  
 2. 答案必須寫在答案卷上，否則不予計分。  
 3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題  
共壹頁  
第壹頁

1. For the nonhomogeneous equation

$$x^2 \frac{d^2 y}{dx^2} - 6x \frac{dy}{dx} + 12y = 16 \ln(x).$$

Please find

- (a) the homogeneous general solution  $y_h$ , and [10%]  
 (b) the homogeneous particular solution  $y_p$ . [10%]

2. Please use Laplace Transformation to solve the initial value problem [20%]

$$y'' + 5y' + 6y = f(t); \quad y(0) = y'(0) = 0;$$

with

$$f(t) = \begin{cases} -2 & \text{for } 0 \leq t < 3, \\ 0 & \text{for } t \geq 3. \end{cases}$$

3. Please find the eigenvalues and eigenvectors of [20%]

$$A = \begin{pmatrix} -3 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 0 \end{pmatrix},$$

where all the eigenvectors are of length 1.

4. Please solve the Dirichlet problem [20%]

$$\begin{aligned} \nabla^2 u(x, y) &= 0 & \text{for } 0 < x < L, \quad 0 < y < K, \\ u(x, 0) &= 0 & \text{for } 0 \leq x \leq L, \\ u(0, y) &= u(L, y) = 0 & \text{for } 0 \leq y \leq K, \\ u(x, K) &= (L - x) \sin(x) & \text{for } 0 \leq x \leq L. \end{aligned}$$

5. Please find the Fourier coefficients and the Fourier series of function
- $f(x) = e^{-4x}$
- for
- $-2 \leq x \leq 2$
- . [20%]