

科目：電磁學 適用：電機所系統組

編號：433

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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Electromagnetics

April 9, 2006

1. A TEM wave propagates within a coaxial structure shown in Fig. 1 with the fields given by

$$\mathbf{E} = \hat{\rho} E_{\rho} = \hat{\rho} \frac{E_0}{\rho} e^{-j\beta z}$$

$$\mathbf{H} = \hat{\phi} H_{\phi} = \hat{\phi} \frac{H_0}{\rho} e^{-j\beta z}$$

Perfect conductors and lossless medium in between are assumed. Derive in details the time-harmonic transmission-line equations for V and I and the expressions for the associated line capacitance (C) and line inductance (L) of this coaxial structure through the two *Curl* equations of the time-harmonic Maxwell's equations. The *Curl* operation in general orthogonal coordinates is given below

$$\nabla \times \mathbf{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} \hat{u}_1 h_1 & \hat{u}_2 h_2 & \hat{u}_3 h_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix},$$

where h_1 , h_2 , and h_3 are the metric coefficients of the coordinates (u_1, u_2, u_3) . (20%)

2. An element shown in Fig. 2 is defined by the following surfaces:

- $r = r_1$ and $r = r_2$,
- $\theta = \frac{\pi}{2}$ and $\theta = \beta$,
- $\phi = 0$ and $\phi = \alpha$.

Compute the following if the material of the element is characterized by a conductivity of σ :

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- (a) the resistance of this element if the surface at $r = r_1$ has $V = 0$ and the surface at $r = r_2$ has $V = V_0$. Then, determine the inductance of this element if the material is characterized by a permeability of μ . Neglect fringing. (10%)
- (b) the resistance of this element if the surface at $\theta = \frac{\pi}{2}$ has $V = 0$ and the surface at $\theta = \beta$ has $V = V_0$. Neglect fringing. (5%)
- (c) the resistance of this element if the surface at $\phi = 0$ has $V = 0$ and the surface at $\phi = \alpha$ has $V = V_0$. Neglect fringing. (5%)
3. For two quarter circular line charges of density $2\rho_\ell$ and $-\rho_\ell$, respectively, located on the $x-y$ plane, as shown in Fig. 3, determine the following quantities at any point $(0, 0, z)$ on the z -axis.
- (a) the electric potential V . (5%)
- (b) the electric field intensity \mathbf{E} . (10%)
4. The electromagnetic fields in a rectangular waveguide shown in Fig. 4 are given by
- $$\mathbf{E} = C \frac{\omega \mu_0 a}{\pi} \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z) \hat{y}$$
- $$\mathbf{H} = -C \frac{\beta a}{\pi} \sin\left(\frac{\pi x}{a}\right) \sin(\omega t - \beta z) \hat{x} + C \cos\left(\frac{\pi x}{a}\right) \cos(\omega t - \beta z) \hat{z}$$
- where C is a constant and $\omega = 2\pi f$, with f the frequency of excitation. The walls of the waveguide are assumed to be perfect conductors. Determine the surface charge densities and surface current densities on those walls. (15%)
5. Determine the following fields at a remote location $P(r, \theta, \phi)$:
- (a) the electric field intensity \mathbf{E} caused by an electric dipole defined by its dipole moment $\mathbf{p} = qd\hat{z}$, as shown in Fig. 5(a), (5%)
- (b) the magnetic field intensity \mathbf{B} caused by a magnetic dipole defined by its dipole moment $\mathbf{m} = IS = I\pi a^2 \hat{z}$, as shown in Fig. 5(b). (10%)

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6. Consider an electromagnetic wave at oblique incidence on a dielectric interface, as shown in Fig. 6. Given the following conditions:

- $\epsilon_1 = 4\epsilon_0$ and $\epsilon_2 = \epsilon_0$,
- $\mu_1 = \mu_2 = \mu_0$,

determine the following:

- (a) the critical angles, θ_c , for both polarizations, (5%)
- (b) the angles of refraction of the refracted waves for both polarizations under the condition of $\theta_i > \theta_c$, (5%)
- (c) and the Brewster angles, θ_B , for both polarizations. (5%)

You may express your answers in terms of the inverse trigonometric functions.

Laplace's equation in cylindrical coordinates:

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0,$$

and in spherical coordinates:

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2} = 0.$$

Some useful integrals:

$$\begin{aligned} \int \sin x dx &= -\cos x + C, & \int \cos x dx &= \sin x + C, \\ \int \tan x dx &= -\ln(\cos x) + C, & \int \cot x dx &= \ln(\sin x) + C, \\ \int \sec x dx &= \ln \left[\tan \left(\frac{\pi}{4} + \frac{x}{2} \right) \right] + C, & \int \csc x dx &= \ln \left(\tan \frac{x}{2} \right) + C, \end{aligned}$$

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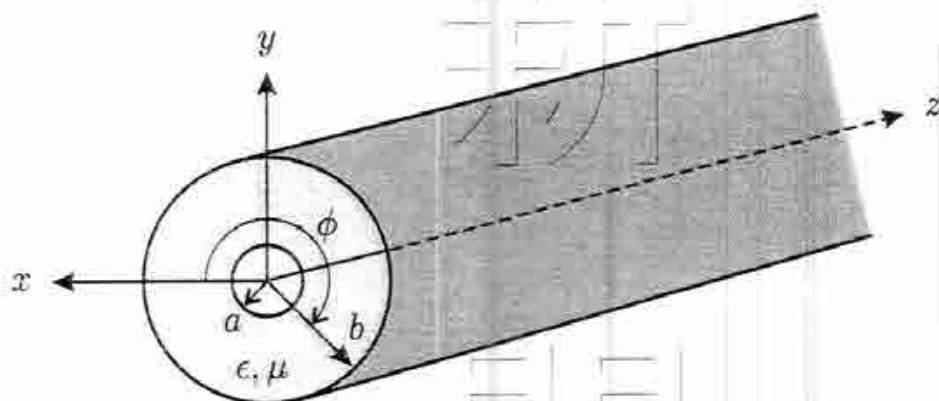


Figure 1: Figure for Problem 1.

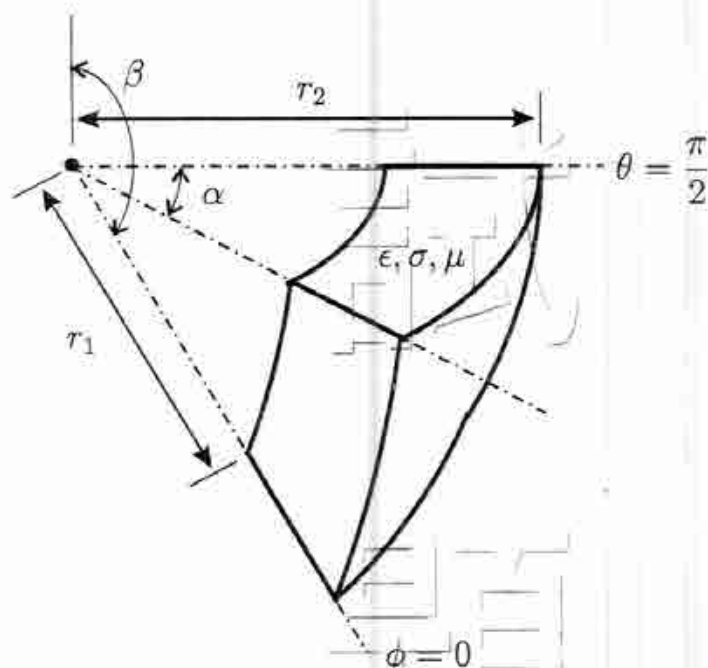


Figure 2: Figure for Problem 2.

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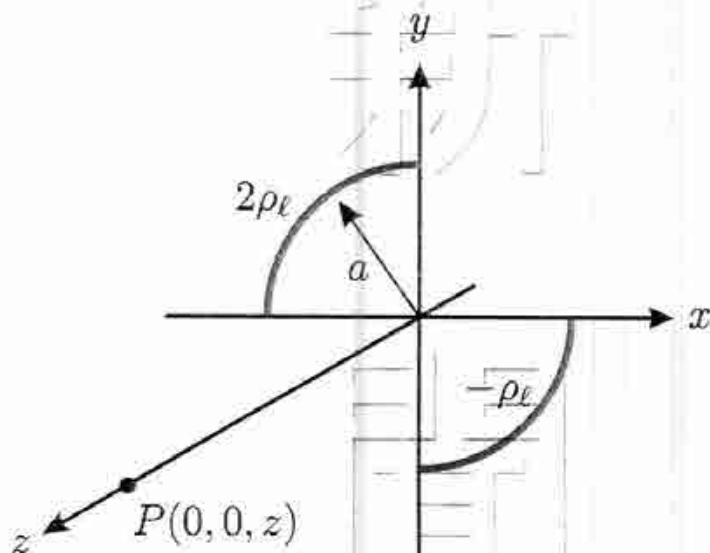


Figure 3: Figure for Problem 3.

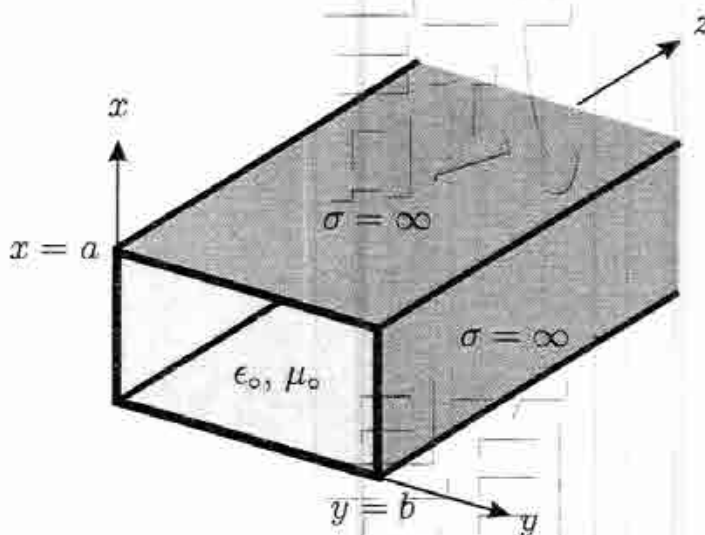


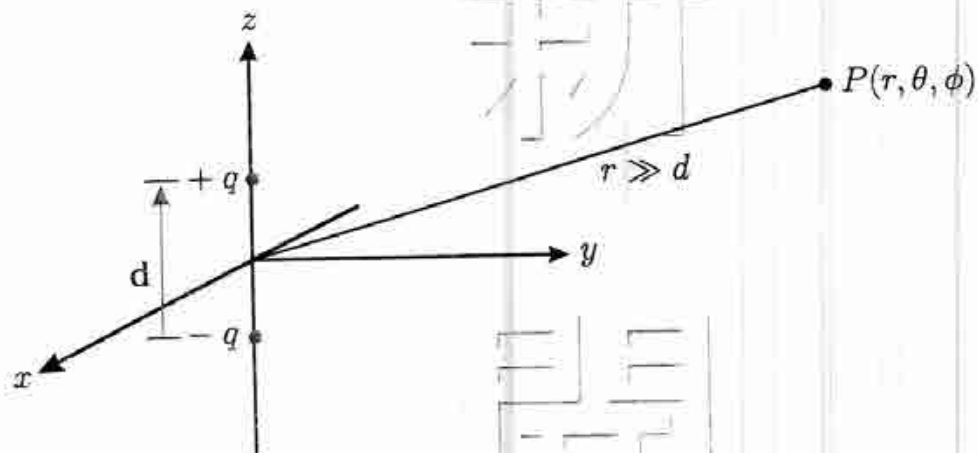
Figure 4: Figure for Problem 4.

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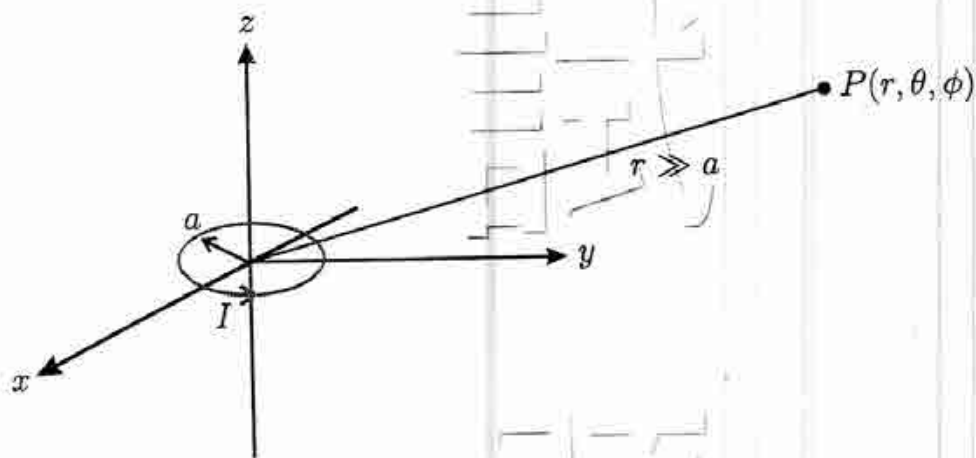
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5(a)



5(b)

Figure 5: Figures for Problem 5.

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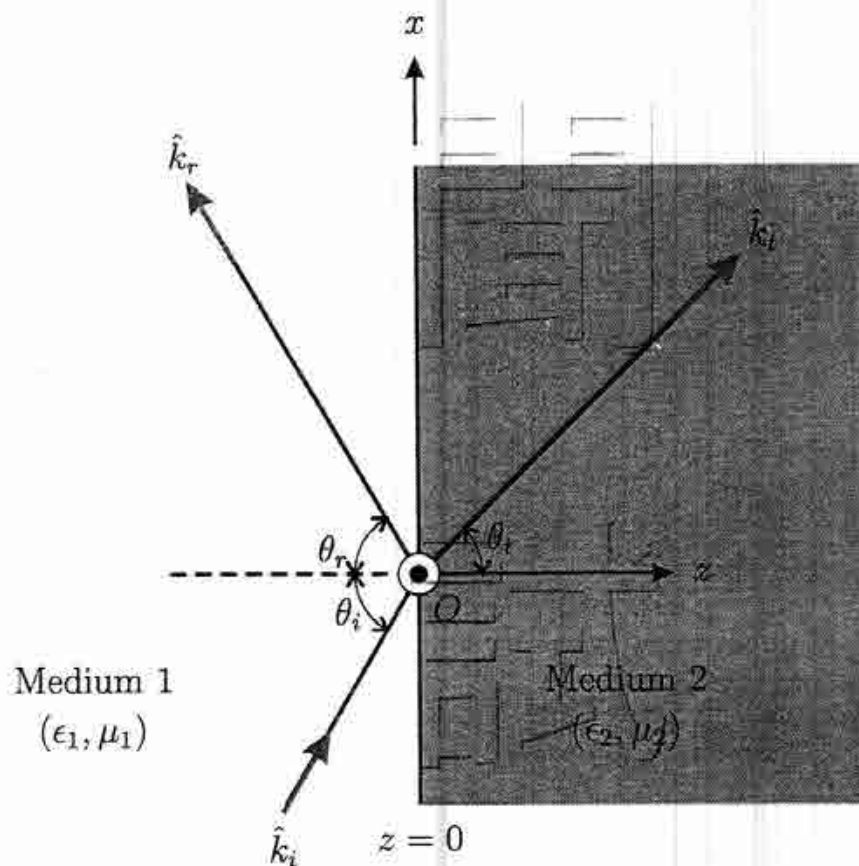
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Figure 6: Figure for Problem 6.