

科目：統計學 適用：財金所

編號：353

考生注意：

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2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題

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第1頁

填充題：每一空格 2%，共 50%，空格寫答案即可，不需有過程

一、If  $X \sim N(10, 36)$ ,  $P(x < 7) = ?$ ,  $P(8 \leq x < 12) = ?$ ,  $P(x = 14) = ?$ .

Answer: 1: \_\_\_\_\_, 2: \_\_\_\_\_, 3: \_\_\_\_\_.

二、The Asian currency crisis of late 1997 and early 1998 was expected to lead to substantial job losses in the United States as inexpensive imports flooded markets. California was expected to be especially hard hit. The Economic Policy Institute estimated that the mean number of job losses in California would be 126,681. Assume the number of jobs lost in California is normally distributed with a standard deviation of 30,000. What is the probability the number of loss jobs is between 80,000 and 150,000?

Answer: 4: \_\_\_\_\_.

三、A good point estimator need to satisfy the following properties:

Answer: 5: \_\_\_\_\_, 6: \_\_\_\_\_, and 7: \_\_\_\_\_.

四、A simple random sample with  $n=24$  provide a sample mean of 22.5 and a sample standard deviation of 4.4. What is a 95% confidence interval for the population mean?

Answer: 8: \_\_\_\_\_.

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第 2 頁

五、A project director specified a desired margin of error of  $E=2$ , and 95% level of confidence. If the sample standard deviation was 9.65, the project needs how many sample size to satisfy the director's objection.  
Answer: 9: \_\_\_\_\_

六、According to Barron's 1998 Primary Reader Survey, the average annual number of investment transactions for a subscriber is 30. Suppose the number of transactions in a year follows the Poisson probability function.

a. What is the probability of no transactions during the month of January for a particular subscriber?

Answer: 10: \_\_\_\_\_

b. What is the probability that the next transaction will occur within the next half month for a particular subscriber? ( $e^{1.25} = 3.49034$ )

Answer: 11: \_\_\_\_\_

七、The prior probabilities of events  $A_1$ ,  $A_2$  and  $A_3$  are  $P(A_1)=0.2$ ,  $P(A_2)=0.5$  and  $P(A_3)=0.3$ . The conditional probabilities of event  $B$  given  $A_1$ ,  $A_2$  and  $A_3$  are  $P(B|A_1)=0.5$ ,  $P(B|A_2)=0.4$  and  $P(B|A_3)=0.3$ .

What are the value of  $P(A_1|B)$ ,  $P(A_2|B)$  and  $P(A_3|B)$ ?

Answer: 12: \_\_\_\_\_, 13: \_\_\_\_\_, 14: \_\_\_\_\_

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八、The following data are the monthly salaries  $y$  and the grade point averages  $x$  for students who obtained a bachelor's degree in business administration with a major in information systems.

GPA	2.6	3.4	3.6	3.2	3.5	2.9
Salary	3300	3600	4000	3500	3900	3600

a. What is the estimated regression equation for these data?

Answer: 15: \_\_\_\_\_

b. Compute SST and SSE.

Answer: 16: \_\_\_\_\_, and 17: \_\_\_\_\_

c. What is the sample correlation coefficient?

Answer: 18: \_\_\_\_\_

d. Develop a 95% confidence interval for the mean starting salary for all students with a 3.0 GPA.

Answer: 19: \_\_\_\_\_

e. Develop a 95% prediction interval for the mean starting salary for Joe Heller, a student with a 3.0 GPA.

Answer: 20: \_\_\_\_\_

九、Let the probability density function of two random variables  $X_1$  and

$X_2$  be  $f(x_1, x_2) = \frac{x_1 x_2}{k}$ , for  $x_1 = 1, 2, 3, 4, x_2 = 3, 5$

What is the value of  $P(X_1 X_2 \geq 10)$ ?

Answer: 21: \_\_\_\_\_

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十、Let  $X \sim N(\mu, \sigma^2)$  and  $Y = \ln X$ . What is the expected value and variance of  $Y$ ?

Answer: 22. \_\_\_\_\_, 23. \_\_\_\_\_

十一、Consider the following hypothesis test:  $H_0: \sigma_1^2 = \sigma_2^2$  and  $H_a: \sigma_1^2 \neq \sigma_2^2$ .

a. What is the value of the test statistic if  $n_1 = 21$ ,  $s_1^2 = 8.2$ ,  $n_2 = 26$ ,  $s_2^2 = 4.0$ ?

Answer: 24. \_\_\_\_\_

b. What is your conclusion using  $\alpha = 0.05$ ?

Answer: 25. \_\_\_\_\_

問答及證明題(共 50%)

一、(15%)試寫出下列分配的機率函數及動差母函數：二項分配，卜瓦松分配，超幾何分配，常態分配及指數分配

二、(每一小題 5%，共 10%)

a. 假設隨機變數  $X \sim N(\mu, \sigma^2)$ ， $\bar{x}$  及  $s^2$  分別為其樣本平均數及樣本變異數，試以  $Z$  分配及  $\chi^2$  分配說明  $t$  分配並由說明推論到下列關係

$$\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} \sim t(n-1)$$

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b. 若  $X_1 \sim N(\mu_1, \sigma_1^2)$ ,  $X_2 \sim N(\mu_2, \sigma_2^2)$ , 試以  $\chi^2$  分配說明 F 分配並由說明推論到下列關係

$$\frac{s_1^2 / \sigma_1^2}{s_2^2 / \sigma_2^2} \sim F(r_1, r_2)$$

三、(每一小題 5% 共 25%) 令線性模型為  $y = X\beta + e$ , 其中  $y$  為  $T \times 1$  的向量,  $X$  為  $T \times k$  的矩陣,  $\beta$  為  $k \times 1$  的向量,  $e$  為  $T \times 1$  的向量, 回答以下各題：

a. 令  $b$  為  $\beta$  的估計值, 請以矩陣表示如何用最小平方方法推論  $b$  值。

b. 若  $(X'X)^{-1} = \begin{bmatrix} 11.9148 & -3.674 & -3.9314 \\ -3.674 & 1.35 & 1.1274 \\ -3.9314 & 1.1214 & 1.353 \end{bmatrix}$ ,  $X'y = \begin{bmatrix} 39.71 \\ 34.2946 \\ 87.2988 \end{bmatrix}$ ,

該模型的估計式為何？

c. 證明  $b$  的變異-共變異矩陣為  $\sigma^2 E[(X'X)^{-1}]$ , 其中  $\sigma^2 = E[e'e]$  為變異數,  $E[\cdot]$  表期望值。

d. 證明估計殘差  $\hat{e} = (I - X(X'X)^{-1}X')e$ 。

e. 證明  $b$  及  $\hat{\sigma}^2$  互相獨立, 其中  $\hat{\sigma}^2 = \frac{\hat{e}'\hat{e}}{T-k}$ 。

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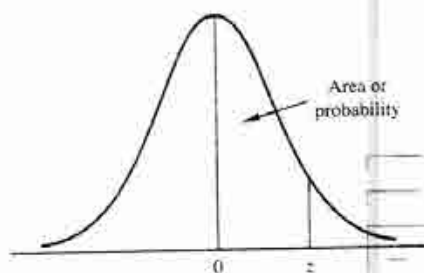
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TABLE 1 STANDARD NORMAL DISTRIBUTION



Entries in the table give the area under the curve between the mean and  $z$  standard deviations above the mean. For example, for  $z = 1.25$  the area under the curve between the mean and  $z$  is .3944.

$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4254	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

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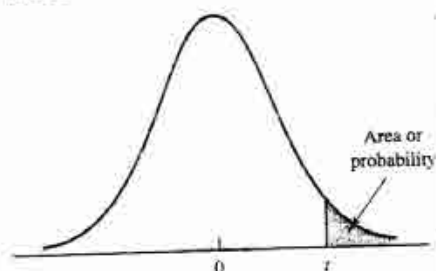
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Appendix B Tables

TABLE 2 *t* DISTRIBUTION

Entries in the table give *t* values for an area or probability in the upper tail of the *t* distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail,  $t_{.05} = 1.812$ .

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756
30	.854	1.310	1.697	2.042	2.457	2.750
31	.853	1.309	1.696	2.040	2.453	2.744
32	.853	1.309	1.694	2.037	2.449	2.738
33	.853	1.308	1.692	2.035	2.445	2.733
34	.852	1.307	1.691	2.032	2.441	2.728

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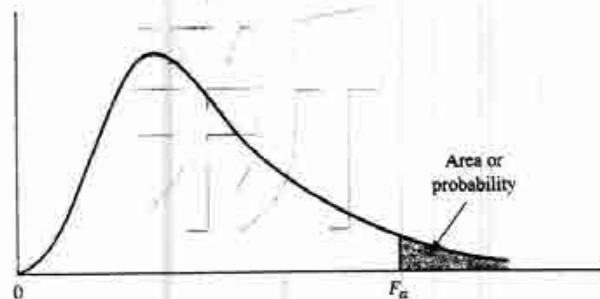
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F DISTRIBUTION



Denominator Degrees of Freedom	Area in Upper Tail	Numerator Degrees of Freedom													
		1	2	3	4	5	6	7	8	9	10	15	20	25	30
16	.10	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.94	1.89	1.86	1.84
	.05	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.35	2.28	2.23	2.19
	.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.79	2.68	2.61	2.57
	.01	8.53	6.23	5.29	4.72	4.44	4.26	4.05	3.89	3.78	3.69	3.41	3.26	3.16	3.10
17	.10	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.91	1.86	1.83	1.81
	.05	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.31	2.23	2.18	2.15
	.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.72	2.62	2.55	2.50
	.01	8.40	6.11	5.19	4.62	4.34	4.16	3.95	3.79	3.68	3.59	3.31	3.16	3.07	3.00
18	.10	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.89	1.84	1.80	1.78
	.05	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.27	2.19	2.14	2.11
	.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.67	2.56	2.49	2.44
	.01	8.29	6.01	5.09	4.52	4.25	4.07	3.84	3.71	3.60	3.51	3.23	3.08	2.98	2.92
19	.10	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.86	1.81	1.78	1.76
	.05	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.23	2.16	2.11	2.07
	.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.62	2.51	2.44	2.39
	.01	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.15	3.00	2.91	2.84
20	.10	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.84	1.79	1.76	1.74
	.05	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.20	2.12	2.07	2.04
	.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.57	2.46	2.40	2.35
	.01	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.09	2.94	2.84	2.78
21	.10	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.83	1.78	1.74	1.72
	.05	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.18	2.10	2.05	2.01
	.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.53	2.42	2.36	2.31
	.01	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.03	2.88	2.79	2.72
22	.10	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.81	1.76	1.73	1.70
	.05	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.15	2.07	2.02	1.98
	.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.50	2.39	2.32	2.27
	.01	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	2.98	2.83	2.73	2.67
23	.10	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.80	1.74	1.71	1.69
	.05	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.13	2.05	2.00	1.96
	.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.47	2.36	2.29	2.24
	.01	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	2.93	2.78	2.69	2.62
24	.10	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.78	1.73	1.70	1.67
	.05	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.11	2.03	1.97	1.94
	.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.44	2.33	2.26	2.21
	.01	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	2.89	2.74	2.64	2.58
25	.10	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.77	1.72	1.68	1.66
	.05	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.09	2.01	1.96	1.92
	.025	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.41	2.30	2.23	2.18
	.01	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.85	2.70	2.60	2.54
26	.10	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.76	1.71	1.67	1.65
	.05	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.07	1.99	1.94	1.90
	.025	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.39	2.28	2.21	2.16
	.01	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.81	2.66	2.57	2.50
27	.10	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.75	1.70	1.66	1.64
	.05	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.06	1.97	1.92	1.88
	.025	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.36	2.25	2.18	2.13
	.01	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.78	2.63	2.54	2.47