

科目：離散數學 適用：資工所

編號：413

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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The following problems may be answered in Chinese or English. You need to give all details in order to receive any credit (point).

Problem 1 (15%) Let $G_0 = 3, G_1 = 4, G_{n+1} = G_n + G_{n-1}$ for $n \geq 1$. Prove that $\sum_{k=0}^n G_k = G_{n+2} - 4$ for $n \geq 5$.

Problem 2 (10%) Prove that there exist k consecutive composite integers for every positive integer k .

Problem 3 (15%) Prove that a graph with n nodes and more than $\binom{n-1}{2}$ edges is always connected. (Note: The graph here is simple and undirected.)

Problem 4 (20%) A graph G is called *planar* iff it can be drawn in the plane with its edges intersecting only at vertices of G . Let G be the graph obtained by omitting an edge of the complete graph K_5 on five nodes. Prove or disprove that G is planar.

Problem 5 Let $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$. Let $P(f, x_0)$ be the statement: for every $\epsilon > 0$, there exists $\delta > 0$ such that $|f(x) - f(x_0)| < \epsilon$ for all $|x - x_0| < \delta$. Here the universe comprises the positive real numbers.

- a. (10%) Express $P(f, x_0)$ by the first-order logic.
- b. (10%) Describe its negating statement by the first-order logic.

Problem 6 (20%) Let f, g and h be functions from reals to reals. Prove that $(f \circ g) \circ h = f \circ (g \circ h)$ where \circ is the composition of two functions.