

科目：數學 (以離散數學、線性代數為主)

編號：343

適用：資工系

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

共 2 頁

第 1 頁

(以下各題均須寫出計算或證明過程方予計分)

1. (6%) List all the permutations for the letters a, c, t.
2. (6%) Determine the value: $\sum_{i=1}^6 (i^2 + 1)$.
3. (10%) Let $n \in \mathbf{Z}^+$, and $n \geq 14$, prove that n can be written as a sum of 3's and/or 8's.
4. (8%) Let $f, g: \mathbf{Z}^+ \rightarrow \mathbf{Z}^+$ where for all $x \in \mathbf{Z}^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x - 1\}$, the maximum of 1 and $x - 1$.
 - (a) (1%) Is f an onto function?
 - (b) (1%) Is the function f one-to-one?
 - (c) (1%) Is g an onto function?
 - (d) (1%) Is the function g one-to-one?
 - (e) (2%) Determine $(f \circ g)(x)$ for $x = 1$, and 7.
 - (f) (2%) Determine $(g \circ f)(x)$ for $x = 1$, and 7.
5. (10%) Find the general solution for the recurrence relation
$$a_{n+3} - 3a_{n+2} + 3a_{n+1} - a_n = 3 + 5n, n \geq 0$$
6. (10%) Prove that for every group G , if $a, b, c \in G$ and $ab = ac$, then $b = c$.

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7. (15%) Let $v_1 = (0, 1, 0)$, $v_2 = (-4/5, 0, 3/5)$, and $v_3 = (3/5, 0, 4/5)$.
- (a) (7%) Show that $S = \{v_1, v_2, v_3\}$ is an orthonormal basis for R^3 with the Euclidean inner product.
- (b) (8%) Express the vector $u = (1, 1, 1)$ as a linear combination of the vectors in S , and find the coordinate vector $(u)_S$.
8. (5%) Find vectors x any y in R^2 that are orthonormal with respect to the inner product $\langle u, v \rangle = 3u_1v_1 + 2u_2v_2$, where $u = (u_1, u_2)$ and $v = (v_1, v_2)$, but are not orthonormal with respect to the Euclidean inner product.
9. (8%) Find the dimension and a basis for the null space of A , where
- $$A = \begin{bmatrix} 3 & 1 & 1 & 1 \\ 5 & -1 & 1 & -1 \end{bmatrix}.$$
10. (7%) Prove that if A is a matrix with n columns, then $\text{rank}(A) + \text{nullity}(A) = n$.
11. (15%) Let $T: R^2 \rightarrow R^2$ be the linear operator given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ 2x_1 + 4x_2 \end{bmatrix}$.
- (a) (4%) Find the standard matrix A for T .
- (b) (6%) Find a basis for R^2 relative to which the matrix for T is diagonal.
- (c) (5%) Find A^{10} .