

科目：工程數學(線性代數、微分方程)

適用：電機

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題
共 2 頁
第 1 頁

系

編號：342

線性代數，50%

1. (15%) Given that $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 2 & -6 \\ 1 & -1 & 2 \end{bmatrix}$, find the rank of A , B and AB .
2. (20%) True or false: (A and B are square matrices of the same size)
- (a) (5%) If A is not invertible then AB is not invertible.
 - (b) (5%) The determinant of A is always the product of its pivots.
 - (c) (5%) The determinant of $A - B$ equals $\det(A) - \det(B)$.
 - (d) (5%) AB and BA have the same determinant.
3. (15%) True or false:
- (a) (3%) If 0 is an eigenvalue of a matrix A , then A is not invertible.
 - (b) (3%) Similar matrices always have the same eigenvectors.
 - (c) (3%) If A^T (the transpose of A) has an eigenvalue λ , then A also has λ as an eigenvalue.
 - (d) (3%) If all the eigenvalues of a square matrix A are zero, then $A = 0$.
 - (e) (3%) If v_1 and v_2 are eigenvectors corresponding to distinct eigenvalues of a matrix A , then $v_1 + v_2$ is also an eigenvector of A .

試

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本 試 題
共 2 頁
第 2 頁

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4. (15%) Solve the following ODE, by letting $z = xy$.

$$xe^{xy}y' + e^{xy}y - 3 = 0.$$

- (a) List the new ODE, after changing the variable. (5%)

- (b) Find the general solution of the given ODE. (10%)

[提示：變數轉換後，求解]

5. (15%) One solution of the following ODE is x^{-4} .

$$y'' = -\left(\frac{9}{x}y' + \frac{A}{x^2}y\right).$$

- (a) Find the value of A . (5%)

- (b) Find the general solution of the ODE. (10%)

6. (20%) Find the series solution of ODE $y'' = -3xy' + y$.

- (a) Find the recursion formula for the coefficients. [提示：求出係數之間的遞迴關係式] (10%)

- (b) Find the solution of the ODE. The solution can have only four non-zero terms. [提示：求出前四項非零項] (10%)

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