

科目：微積分 適用：資工系二、土木系二、應化系二、電機系二、
應光系二

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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(以下各題均須寫出計算過程方予計分)

1. (15%) Definition A: A function f is said to be *differential* at x iff

$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ exists. If this limit exists, it is called the *derivative of f at x* ,

and is denoted by $f'(x)$. Let $f(x) = \frac{1}{x}$.

- (a) (5%) Sketch the graph of the function $f(x)$.
 - (b) (5%) According to the Definition A, show that $f'(x) = -\frac{1}{x^2}$.
 - (c) (5%) Find the tangent line that passes through the point $(2, 1/2)$ with slope $f'(2)$.
2. (10%) Does there exist a differentiable function f that satisfies $f(0) = 2, f(2) = 5$, and $f'(x) \leq 1$ on $(0, 2)$? If not, why not?

3. (15%) Let f be continuous on $[a, b]$.

- (a) (10%) Then, the function F defined on $[a, b]$ by $F(x) = \int_a^x f(t) dt$ is continuous on $[a, b]$ and differentiable on (a, b) . Prove that $F'(x) = f(x)$ for all x in (a, b) . Hint: $F(x+h) - F(x) \approx f(x) \times h$ if h is small enough. (圖示證明(pictorial proof)即可)

- (b) (5%) Let $F(x) = \int_{x^2}^1 (t - \sin^2 t) dt$, find $F'(x)$.

4. (10%) Evaluate the integral $\int_0^{1/2} (\cos^3 \pi x) \sin \pi x \, dx$

5. (10%) Show that the improper integral $\int_1^4 \frac{dx}{(x-2)^2}$ diverges.

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6. (10%) Find the Maclaurin series for $f(x) = \frac{1}{\sqrt{4-x}}$ and its radius of convergence.
7. (10%) Let $g(x, y) = \frac{x^2 y}{x^4 + y^2}$. Show that $\lim_{(x, y) \rightarrow (0, 0)} g(x, y)$ does not exist.
8. (10%) Find the linear approximation of the function $f(x, y) = \sqrt{20 - x^2 - 7y^2}$ at $(2, 1)$ and use it to approximate $f(1.95, 1.08)$.
9. (10%) Find the directional derivative of the function $f(x, y, z) = x \cos y \sin z$ at the point $(1, \pi, \pi/4)$ in the direction of the vector $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$.