

科目：工程數學(線性代數+微分方程) 適用：電機系三

編號：842

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

共 1 頁

第 1 頁

## 線性代數

1. (15%) If  $A$  is a  $2 \times 2$  matrix with the determinant  $|A| = 5$ , compute the following determinants.

- (a) (3%)  $|3A|$ , (b) (3%)  $|A^2|$ , (c) (3%)  $|2A^T|$ , (d) (3%)  $|3A|$ , (e) (3%)  $|5A^{-1}|$ .

2. (10%) Find the eigenvalues and corresponding eigenvectors of the matrix  $A =$

$$\begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}.$$

3. (15%) Find the dimension of the following vector spaces.

- (a) (3%) The plane  $x - 3y + 2z = 0$  in  $\mathbb{R}^3$ .  
 (b) (3%) The set of all  $3 \times 3$  diagonal matrices with real elements.  
 (c) (3%) The set of all  $3 \times 3$  upper triangular matrices with real elements.  
 (d) (3%) All vectors of the form  $(a, b, c, 0)$  in  $\mathbb{R}^4$ .  
 (e) (3%) All real-coefficient polynomials of the form  $ax^3 + bx^2 + cx + d$  with  $a = 3d$ .

4. (10%) Find the nullity and rank of the matrices  $A$  and  $B$ , where

$$A = \begin{bmatrix} 1 & 2 & -1 & 2 \\ 3 & 4 & 1 & 6 \\ 5 & 4 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 2 & 5 & 8 \end{bmatrix}.$$

## 微分方程

1. (15%) Solve each of the following differential equations.

- (a) (5%)  $(6xy - y^3)dx + (4y + 3x^2 - 3xy^2)dy = 0$ .  
 (b) (5%)  $2y'' - 7y' + 3y = 0$ .  
 (c) (5%)  $y' - 4y' = 2e^{2x}$ .

2. (15%) Solve each of the following initial value problems.

- (a) (5%)  $y'' + 5y' + 6y = 0$ ;  $y(0) = 0, y'(0) = 1$ .  
 (b) (5%)  $2y^{(3)} - 3y'' - 2y' = 0$ ;  $y(0) = 1, y'(0) = -1, y''(0) = 3$ .  
 (c) (5%)  $y'' + 4y = 2x$ ;  $y(0) = 1, y'(0) = 2$ .

3. (10%) Find the inverse Laplace transform of the function  $F(s)$ .

- (a) (5%)  $F(s) = \frac{1}{s(s^2 + 4)}$ .  
 (b) (5%)  $F(s) = \frac{1}{s(s+1)(s+2)}$ .

4. (10%) Use Laplace transform to solve the initial value problem:

$$y'' - y' - 2y = 0; y(0) = 0, y'(0) = 2.$$