

科目：離散數學

適用：資工系三

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

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(以下各題均須寫出計算或證明過程方予計分)

1. Determine the coefficient of

(a) x^9y^3 in $(x+y)^{12}$ (5%)

(b) xyz^2 in $(2x-y-z)^4$ (5%)

(c) $w^2x^2y^2z^2$ in $(2w-x+3y+z-2)^{12}$ (5%)

2. Find the digits x, y, z where $(xyz)_9 = (zyx)_6$. (15%)3. Prove that if we select 101 integers from the set $\{1, 2, 3, \dots, 300\}$, there exist m, n in the selection where $\gcd(m, n) = 1$. (10%)

4. Find the number of ways to arrange the letters in LAPTOP so that none of the letters L, A, T, O is in its original position and the letter P is not in the third or sixth position. (15%)

5. In how many ways can four of the letter in ENGINE be arranged? (10%)

6. Solve the following recurrence relations.

(a) $2a_{n+2} - 11a_{n+1} + 5a_n = 0, n \geq 0, a_0 = 2, a_1 = -8$. (10%)

(b) $a_{n+2} + 4a_{n+1} + 4a_n = 7, n \geq 0, a_0 = 1, a_1 = 2$. (10%)

7. Let $G = (V, E)$ be a loop-free undirected graph. If $\deg(v) \geq 2$ for all $v \in V$, prove that G contains a cycle. (15%)