

科目：工程數學(線性代數+微分方程)

編號：342 適用：電機系

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題

共 2 頁

第 1 頁

(一) (32 分, 每小題 4 分) Consider $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 5 \\ 4 & 1 & k \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 8 \\ m \end{bmatrix}$;

(a) If $k=13$, Is matrix A is nonsingular?(b) Find the eigenvalues of matrix A ?(c) Same as (b), Find the corresponding normalized eigenvectors for matrix A (norm equals to 1)?(d) If $\det(A) = -9$, Find k ?(e) Same as (d), Find A^T ?(f) If A is not invertible, Find k ?(g) Same as (f), linear system $Ax=B$ has at least one solution, Find m ?(h) Same as (g), Find solution of x

(二) (18 分, 每小題 3 分) Consider the vector space of all functions of a variable x . Determine the following sets of functions are linear independent or linear dependent .(show details)

(a) $\cos x, x \cos x$ (b) $x^2-2x+5, x^2-4x+10$ (c) $\sin^2 x, \cos^2 x, \cos 2x$ (d) $x^2-2x+5, x^2-5x+10, x^2$ (e) $\sin x, \sin^2 x, \cos^2 x, 1$ (f) $e^x, e^{2x}, e^{3x}, \dots, e^{nx}, \dots$

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(三) One solution of the ODE $xy'' - y' - 4x^3y = 0$ is e^{x^A} , where A is a constant.

- (a) Find the constant A . (5%)
- (b) Find the general solution. (10%)

(四) Consider the ODE with the constant A

$$y' - \frac{3x^2y}{(1-x^3)} = A, \quad y(0) = 2, \quad y(-1) = -3/2.$$

- (a) Find the constant A . (5%)
- (b) Solve the ODE. (10%)

(五) (20%) Use the Laplace transform to solve the ODE

$$(t-1)y'' + (2-t)y' + 2y = 1, \quad y(1) = 2.$$

Hint: 1. Please use the Laplace transform method. 請使用 Laplace 轉換法求解。

2. Note that the value of $y(t)$ at $t = 1$ is given.