

科目：工程數學(線性代數+微分方程)

編號：342 適用：電機系

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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(一) (15%, 每小題 5 %) For the linear system $Ax = b$,

$$\text{where } A = \begin{bmatrix} 0 & 1 & 1 & 3 & 4 \\ 1 & -2 & 1 & 1 & 2 \\ 1 & 2 & 5 & 13 & 5 \\ -1 & 3 & 0 & 2 & -2 \end{bmatrix}$$

- (a) Use Gaussian elimination procedure to find the reduced row echelon form of A .
- (b) Find the rank of A and a basis for the column space of A .
- (c) Find a basis for the nullspace $N(A)$. What is the dimension of $N(A)$?

(二) (20%, 每小題 5 %) Consider the following Linear system equation

$$x_1 + x_2 + ax_3 = 2$$

$$x_1 + ax_2 + x_3 = 6$$

$$x_1 + x_2 + ax_3 = 4a$$

Determine a values to make system with

- (a) A unique solution
- (b) No solution
- (c) Infinitely many solutions
- (d) Use Cramer's rule to find x_1, x_2, x_3 , if the system has a unique solution

(三) (15%, 每小題 5 %) For matrix $A =$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 6 & 4 & 2 \end{bmatrix}$$

- (a) Find the eigenvalues and corresponding normalized eigenvectors (norm equals to 1) for matrix A .
- (b) Find the transpose matrix A^T .
- (c) Find the eigenvalues and corresponding normalized eigenvectors (norm equals to 1) for matrix A^T .

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(四) (15%) Assume that $(y + A)x^B$ is one integrating factor of the ODE $3(y + 1)dx - 2xdy = 0$. Find the constants A and B , and solve the ODE.

(五) (15%) Consider that $3 \cos 2x$ and $(5 \sin 2x + 7 \cos 2x)$ are the solutions of the ODE $Ay'' + By = 0$, where A and B are two constants. Find the constants A and B , and solve the ODE $Ay'' + By = 4 \cos 2x$.

(六) (20%) Solve the following system of ODE

$$\begin{cases} y_1' = y_1 + 3e^t \\ y_2' = -y_2 + 2e^t \end{cases}$$

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