國立暨南國際大學九十二學年度轉學生入學考試試題

第 2 節微積分適用:(土木系二 321)

(本試題共 1 頁，第 1 頁)

考生注意: 1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. Evaluate the limits: (3 pts for each question, total 15 pts)

(a) $\lim_{x\to 0}\frac{x}{|x|}$ (b) $\lim_{x\to 0}(\frac{1-\cos x}{x^2})$ (c) $\lim_{x\to 0}(x\cot x)$ (d) $\lim_{(x,y)\to(0,0)}\frac{x^2y}{x^2+y^2}$

(e) $\lim_{t\to 0}\left\langle e^{2t}+5,\ \ t^2+2t-4,\ \ \frac{1}{t}\right\rangle$

2. Find the first derivative of y or $\vec{y}$: (3 pts for each question, total 15 pts)

(a) $y=\sin(\frac{2x}{x+1})$ (b) $y=\frac{x^2-2}{x^2+1}$ (c) $y=\int_2^{x^2}\cos t\,dt$ (d) $\sin(xy)+x^2=x-y$

(e) $\vec{y}(t)=\left\langle\frac{t-3}{t+1},\ \ te^{2t},\ \ t^3\right\rangle$

3. Evaluate the Integrations: (4 pts for each question, total 20 pts)

(a) $\int_0^2\frac{4x}{x^2+1}dx$ (b) $\int_{-\pi}^{\pi}4x\sec x^2\tan x^2\,dx$ (c) $\int\cos x\ln(\sin x)dx$ (d) $\int\frac{3x+8}{x^3+5x^2+6x}dx$

(e) $\int_0^2\left\langle\frac{4}{t+1},\ \ e^{t-2},\ \ te^t\right\rangle dt$

4. Compute the linear approximation of the function at the given point: (5 pts for each question, total 10 pts)

(a) $f(x)=\sin x$ at $x=0$

(b) $f(x,y)=\sin x\cos y$ at $(0,\pi)$

5. Find the velocity and acceleration vectors for the position vector $\vec{r}(t)=\langle 4\cos 2t,\ \ 4\sin 2t,\ \ 4t\rangle$ at $t=\pi/4$.

(10 pts)

6. Prove that the Taylor expansion of $\sin x$ at $x=0$ is $\sin x=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}x^{2k+1}$ and use it to approximate $\int_{-1}^{1}\frac{\sin x}{x}dx$ with first 3 terms (i.e., n=3). (15 pts)

7. (a) Use Jacobian between Cartesian coordinates and polar coordinates to derive the evaluation formula for polar coordinates: $\iint_R f(x,y)dxdy=\iint_S f(r\cos\theta,r\sin\theta)r\,dr\,d\theta$ (5 pts)

(b) Evaluate $\int_0^2\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}2xy\,dy\,dx$ by converting to polar coordinates using the formula of (a). (10 pts)