國立暨南國際大學九十二學年度轉學生入學考試試題

第 2 節微積分適用:(資管系二 221)

(本試題共 2 頁，第 1 頁)

考生注意: 1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

試題

1. Let $P(1,0)$ be a point on the graph $(x^2+y^2)^2 = x^2y+1$. Fine the equation of the tanget line passing through point P. (15%)

2. Evaluate the following integrals

(i) $\int_{-\infty}^{\infty} e^{(x-e^x)}dx$. (7%)

(ii) $\int_0^8 \frac{1}{1+\sqrt[3]{x}}dx$. (8%)

3. Find the area bounded by the graph $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $a>0$ and $b>0$ (Hint: Formulate this problem as a double integral and use the change of variables by finding the Jacobian matrix). (15%)

4. Maximize $f(x,y,z)=xyz$ subject to the side condition $x^3+y^3+z^3=1$ with $x\geq 0, y\geq 0, z\geq 0$ (Hint: Consider the Lagrange multiplier). (10%)

5. (i) Let $u=f(x_1,x_2,\cdots,x_n)$ be a differentiable function of n variables $x_1,x_2,\cdots,x_n$ and each x_i, $i=1,\cdots,n$, is a function of the m variables $t_1,t_2,\cdots,t_m$ such that all the partial derivatives $\partial x_j/\partial x_i$ exist, $1\leq i\leq m$ and $1\leq j\leq n$. According to the chain rule, write down the expression of $\partial u/\partial t_i$. (5%)

(ii) If $z=2^{x-3y}$, where $x=s^2t$ and $y=st^2$, use your answer in (i) to find $\partial z/\partial s$ and $\partial z/\partial t$ (10%).

6. Derive the following two reduction formulas

(i) $\int x^n \sin x dx = -x^n \cos x + n\int x^{n-1}\cos x dx$. (7%)

(ii) $\int \sin^n x dx = -\frac{1}{n}\sin^{n-1}x\cos x + \frac{n-1}{n}\int \sin^{n-2} x dx$. (8%)

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7. (i) Assume that f can be expressed as a Maclaurin series (a special case of Taylor series). Suppose that a^n is the coefficient of x^n when f is expressed as a Maclaurin series. Show that $f^{(n)}(0) = n! \cdot a_n$ (5%).

(ii) Assume that f can be expressed as a Maclaurin series. If $g(x) = \int_0^x f(t)dt$, show that $g^{(n)}(0) = f^{(n-1)}(0)$ (Hint: use (i)) (10%)