國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 3 節微積分 適用:(財金所 344)

考生注意: 1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. (10%) A company is increasing the production of a product at the rate of 200 units per week. The weekly demand function is modeled by $p = 100 - 0.001x$, where p is the price per unit and x is the number of units produced in a week. Find the rate of change of the revenue with respect to time when the weekly production is 2000 units.

2. (10%) The demand function for a product is modeled by $p = \sqrt{450 - x}$, $0 \le x \le 450$. Find the intervals on which the demand is elastic, inelastic and of unit elasticity, and describe the behavior of the revenue function.

3. (10%) The demand and supply functions for a product are modeled by *Demand:* $p = -0.36x + 9$ and *Supply:* $p = 0.14x + 2$, where x is the number of units (in millions). Find the consumer and producer surpluses for this product.

4. (14%)On a clock, what time between 2 and 3 o'clock will the two hands (one for hour and another for minute) coincide?

5. (16%) Find the indefinite integrals for (a) $\int \frac{x}{(x+1)^2} dx$, (b) $\int xe^x dx$, (c) $\int \frac{x^5 + x - 1}{x^4 - x^3} dx$, (d) $\int \frac{1}{x^2 - 4x + 1} dx$.

6. (10%) A manufacturer determines that the profit for selling x units of one product and y units of a second product is $p = -(x-200)^2 - (y-100)^2 + 5{,}000$. The weekly sales for product 1 vary between 150 and 200 units, and the weekly sales for product 2 vary between 80 and 100 units. Estimate the average weekly profit for the two products.

7. (10%) Prove the quotient rule, i.e., $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$.

8. (10%) Find the intervals on which the function is continuous (a) $f(x) = \frac{1}{(x+4)^2}$, (b) $f(x) = \frac{3}{(x+1)}$, (c) $f(x) = [x+3]$, (d) $f(x) = \begin{cases} x & x \le 0 \\ x+1 & x > 0 \end{cases}$.

9. (10%) Use Newton's Method to approximate the zeros of $f(x) = e^x + x$. Continue the iterations until two successive approximations differ by less than 0.001.