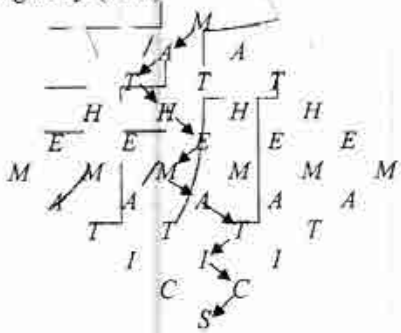
國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 2 節離散數學 適用:(資管所 333)

考生注意: 1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. In how many ways can we spell the word MATHEMATICS by a downward path starting at the top in the following array. (10%)



2. Find the number of primes p ≤ 120. (15%)

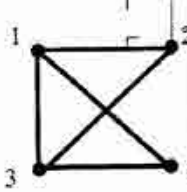
3. The *Fibonacci numbers* F_n have the initial values $F_0 = 0$, $F_1 = 1$, and the recursion $F_n = F_{n-1} + F_{n-2}$ if $n \geq 2$.

   Prove by induction that $F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$ if $n \geq 0$. (15%)

4. Prove that in any list of ten natural numbers, $a_1, a_2, \ldots, a_{10}$, there is a string of consecutive items of the list $a_i + a_{i+1} + a_{i+2} + \ldots$ whose sum is divisible by 10. (15%)

5. Prove that a graph contains an Eulerian cycle if and only if the edges of the graph can be partitioned into cycles. (15%)

6. How many spanning trees does the following graph have? (15%)



7. Prove that a connected graph with n vertices is a tree if and only if it has n -1 edges. (15%)