國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 3 節統計學 適用:(經濟所 323) (本試題共 2 頁，第 1 頁)

考生注意: 1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. Let Y_1 and Y_2 be IID random variables with the density function

$$f(y) = \begin{cases} \frac{1}{\beta}\exp(-\frac{y}{\beta}) & y > 0 \\ 0 & y \leq 0 \end{cases}$$

   Define $U_1 = Y_1 + Y_2$ and $U_2 = \frac{Y_1}{Y_1+Y_2}$. Are U_1 and U_2 independent? (10 points)

2. Suppose the data generating function is: $y_t = \alpha + \beta x_t + e_t$, where x_t is non-stochastic, e_t is IID with mean zero and variance σ^2 and $t = 1, 2, \cdots, n$. The regression model is: $y_t = \gamma x_t + e_t$. Is the LS estimator$(\hat{\gamma})$ unbiased for β? (10 points)

3. Suppose $y_t = \alpha + \beta x_t + \phi y_{t-1} + u_t$, $u_t = \rho u_{t-1} + e_t$, where x_t is nostochastic.

   (a) What is the problem with LS estimators in the regression model mentioned above? (10 points)

   (b) How do you test whether u_t is serially correlated or not? (10 points)

4. Suppose that

$$\begin{aligned} y_1 &= \alpha_1 + \beta_1 x_1 + u_1 \quad \text{for period 1} \\ y_2 &= \alpha_2 + \beta_2 x_2 + u_2 \quad \text{for period 2} \end{aligned}$$

   Describe the procedure to test $H_0 : \beta_1 = \beta_2$. (10 points)

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5. Let X be a uniform distribution on $(0,1)$.

a) Find the density of (10%)

$$Y = -\lambda^{-1}\log(1-X) \text{ for } \lambda > 0.$$

b) Find the mean value of Y. (5%)

6. Let X_1 and X_2 have the joint $p.d.f.$

$f(x_1, x_2) = 2, \qquad 0 < x_1 < x_2 < 1$

a) Find the marginal density function for X_1 and X_2 (5%)

b) Find the conditional $p.d.f.$ of X_1, given $X_2 = x_2$. (5%)

c) Find the conditional mean and conditional variance of X_1, given $X_2 = x_2$. (5%)

c) Compare the values of $\Pr(0 < X_1 < \frac{1}{2} | X_2 = \frac{3}{4})$ and $\Pr(0 < X_1 < \frac{1}{2})$. (5%)

7. Let X have a $p.d.f.$ of the form

$$f(x;\theta) = \frac{1}{\theta}e^{-x/\theta}, \quad 0 < x < \infty$$
$$= 0 \text{ elsewhere,}$$

where $\theta \in \{2,4\}$ to test the simple hypothesis $H_0 : \theta = 2$ against the alternative simple hypothesis $H_1 : \theta = 4$, use a random sample X_1, X_2 of size $n = 2$ and define the critical region to be

$$C = \{(x_1, x_2) : 9.5 \le x_1 + x_2 < \infty\},$$

where X_1, X_2 are identical independent distribution.

Find the power function of test. (15%)