考生注意: 1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。〈餘請詳閱試場規則〉

1. Consider two i.i.d. random variables X and Y with probability density functions (pdf's)

$$f_X(x) = f_Y(x) = \frac{1}{a}\Pi\left(\frac{x}{a}\right),$$

where $\Pi(y)$ denotes the rectangular function which is 1 for $|y| < \frac{1}{2}$, and 0 otherwise. Compute the pdf of $Z = X + Y$ using characteristic functions. (10%)

2. Let $\boldsymbol{X} = (X_1, X_2, X_3)^T$ be a random vector with mean vector $\boldsymbol{\mu} = E\{\boldsymbol{X}\}$ given by

$$\boldsymbol{\mu} = (5, -5, 6)^T$$

and covariance given by

$$\boldsymbol{K} = \begin{bmatrix} 5 & 2 & -1 \\ 2 & 5 & 0 \\ -1 & 0 & 4 \end{bmatrix}$$

Calculate the mean and variance of $Y = \boldsymbol{A}^T\boldsymbol{X} + B$, where $\boldsymbol{A} = (2, -1, 2)^T$ and $B = 5$. (20%)

3. Consider $M-ary$ coherent FSK in terms of signal space. The transmitted signal set is

$$s_i(t) = A\cos\{2\pi[f_c + (i-1)\Delta f]t\}, \qquad 0 \le t \le T_s$$

where

$$\Delta f = \frac{m}{2T_s}, \quad m \text{ is an integer}, \quad i = 1, 2, \cdots, M$$

It is assumed that f_cT_s is an integer. Find an orthonormal basis set and the optimum receiver. Please explain your answer. (20%)

4. A channel of bandwidth 100 kHz is available. Using null-to-null RF bandwidths, what data rates may be supported by: (a) BPSK? (b) Coherent FSK (tone spacing=1/2T)? (c) DPSK? (d) Noncoherent FSK (tone spacing = 2/T)? (20%)

5. A DSSS system employing BPSK data modulation operates with a data rate of 1Mbps. A processing gain of 20 dB is desired.

   (a) Find the required chip rate. (5%)

   (b) What is the RF transmission bandwidth required (null-to-null)? (5%)

6. Consider the set of three finite-energy signals

$$\begin{aligned} s_1(t) &= 1, & 0 \le t \le 1 \\ s_2(t) &= \cos(2\pi t), & 0 \le t \le 1 \\ s_3(t) &= 1 - \sin^2(\pi t), & 0 \le t \le 1 \end{aligned}$$

   Find an orthonormal basis for the signal space spanned by these three signals. (15%)

7. For a binary source, $p(1) = \alpha$ and $p(0) = 1 - \alpha = \beta$. Derive the entropy of the source as a function of α. (5%)