國立暨南國際大學九十二學年度碩士班研究生入學考試試題

（本試題共 頁，第 頁）

第 1 節工程數學 適用：(通訊所電波組 451 通訊所系統組 461)

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(輸請詳閱試場規則)

1. (17%) $X_1, X_2, X_3, X_4, X_5, X_6$ are six iid(independent and identically distributed) exponential random variables each with the following PDF(probability density function) $f_X(x)$.

$$f_X(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$Y_1 = \min\{X_1, X_2, X_3\}$, $Y_2 = \min\{X_4, X_5, X_6\}$, and $Z = \max\{Y_1, Y_2\}$. Find the PDF $f_Z(z)$ of Z.

2. (16%) X and Y are two random variables with the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 1 & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$Z = (X^2 + Y^2)$. Let event $A = \{(X^2 + Y^2) \leq 1\}$. Find the conditional PDF $f_{Z|A}(z)$.

3. (17%) $\underline{\mathbf{X}} = (X_1, X_2, X_3)$, X_k's are Gaussian random variables with $\mathrm{E}[X_k] = 0$, for $k = 1, 2, 3$. The covariance matrix Λ_X is

$$\Lambda_X = E[\underline{\mathbf{X}}^T \underline{\mathbf{X}}] = \begin{bmatrix} 5 & -1 & 2 \\ -1 & 8 & -1 \\ 2 & -1 & 5 \end{bmatrix}.$$

Find a matrix U such that $\underline{\mathbf{Y}} = (Y_1, Y_2, Y_3)$, $\underline{\mathbf{Y}}^T = U\underline{\mathbf{X}}^T$, and Y_1, Y_2, Y_3 are independent Gaussian random variables. (Hint: What is the covariance matrix $\Lambda_Y = \mathrm{E}[\underline{\mathbf{Y}}^T \underline{\mathbf{Y}}]$?)

4. Let $\mathbf{x}^H$ represent the hermitian transpose of vector $\mathbf{x}$ and $\|\mathbf{x}\|_2 = (\mathbf{x}^H \mathbf{x})^{1/2}$, prove

(10%) (a) $|\mathbf{x}^H \mathbf{y}| \leq \|\mathbf{x}\|_2 \|\mathbf{y}\|_2$

(5%) (b) $\|\mathbf{x} + \mathbf{y}\|_2 \leq \|\mathbf{x}\|_2 + \|\mathbf{y}\|_2$

5. Solve the following initial-value problems.

(10%) (a) $x' = -3x - \frac{5}{2}y + \frac{5}{2}\sin 2t$

$y' = 4x + 3y + 2\cos 2t - 3\sin 2t$

$x(0) = -1,\ y(0) = 1.$

(10%) (b) $y'' + 12y' + 32y = f(t),\ y(0) = 0,\ y'(0) = -1$

$$f(t) = \begin{cases} t, & \text{if } 0 \leq t < 1 \\ 2 - t, & \text{if } 1 \leq t < 2 \end{cases}$$

and $f(t) = f(t-2)$ if $t \geq 2$.

6. (15%) Let $P_m(x)$ and $P_n(x)$ be the solutions of Legendre's equations $(1-x^2)y'' - 2xy' + m(m+1)y = 0$ and $(1-x^2)y'' - 2xy' + n(n+1)y = 0$, respectively. Prove that $P_m(x)$ and $P_n(x)$ are orthogonal on the interval $[-1,1]$ by showing that

$$\int_{-1}^{1} P_m(x) P_n(x) dx = 0, m \neq n.$$