國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 2 節離散數學 適用:(資工所 413) (本試題共 / 頁，第 / 頁)

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. True or False ($10 \times 4\%$) (不須寫出計算過程，答錯每題倒扣 4%)

   (1) Let $(A, \Re)$ be a poset. If $(A, \Re)$ is a total order, then it is a lattice.
   (2) All people who are concerned about the environment recycle their plastic containers. Margarita is not concerned about the environment. Therefore Margarita does not recycle her plastic containers.
   (3) $[[(p \vee q) \rightarrow r] \wedge (s \rightarrow \neg p) \wedge (s \wedge \neg q)] \Rightarrow \neg r$.
   (4) $\forall y \in \mathbf{R}, \exists x \in \mathbf{Z}\, (y - x = y + x^2)$.
   (5) $[(A \cup C = B \cup C) \wedge ((A - C = B - C)] \Rightarrow A = B$.
   (6) Let $f, g: \mathbf{Z}^+ \rightarrow \mathbf{Z}^+$ where for all $x \in \mathbf{Z}^+$, $f(x) = x + 1$ and $g(x) = \max\{1, x - 1\}$, the maximum of 1 and $x - 1$. Then $(g \circ f)(x) = (f \circ g)(x) = x$ for all $x \in \mathbf{Z}^+$.
   (7) If $n \in \mathbf{Z}^+$ and $2^n + 1$ is prime, then n is prime.
   (8) If m pigeons occupy n pigeonholes, then at least one pigeonhole has $\lfloor (m-1)/n \rfloor + 1$ or more pigeons roosting in it.
   (9) If $G = (V, E)$ is a forest with $|V| = n$, $|E| = m$, and κ components (trees), then $m = n - \kappa$.
   (10) If F is any field, let $f(x), g(x) \in F[x]$. If $f(x), g(x)$ are relatively prime, then there is no element $a \in F$ with $f(a) = 0$ and $g(a) = 0$.

(以下各題均須寫出計算過程方予計分)

2. (15 %) Let $k \in \mathbf{Z}^+$. Prove that there exists a positive integer n such that $k \mid n$ and the only digits in n are 0's and 3's.

3. (15 %) For $b \in \mathbf{R}^+$, consider the $n \times n$ determinant D_n given by

$$\begin{vmatrix}
b & b & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
b & b & b & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & b & b & b & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & b & b & b & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & \cdots & b & b & b & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & b & b & b & 0 \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & b & b & b \\
0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & b & b
\end{vmatrix}$$

Find the value of D_n as a function of n.

4. (15 %) Let $A = \mathbf{R}^+$. Define addition and multiplication, denoted by $\oplus$ and $\odot$, respectively, on the set A as follows. For $a, b \in A$, $a \oplus b = ab$, the ordinary product of a, b; and $a \odot b = a^{\log_2 b}$.
   (a) Prove that $(A, \oplus, \odot)$ is a ring. (7 %)
   (b) Is this ring commutative? (1 %)
   (c) Does the ring have a unity? What about units? (4 %)
   (d) Is this ring an integral domain or field? (3 %)

5. (15 %) If $G = (V, E)$ is an undirected graph, a subset I of V is called *independent* if no two vertices in I are adjacent. A subset K of V is called a *covering* of G if for every edge $\{a, b\}$ of G either a or b is in K. Prove that if $I \subseteq V$, then I is an independent set in G if and only if $V - I$ is a covering of G.