

國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 1 節工程數學 適用：(電機所系統組 431)

(本試題共 2 頁，第 / 頁)

考生注意：1. 依次序作答，只要標明題號，不必抄題。

2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。

3. 試題隨卷繳回。(餘詳詳閱試場規則)

1. (10%)

$$A = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{3}{4} & 0 \\ -\frac{1}{4} & -\frac{1}{4} & \frac{2}{4} \end{bmatrix}$$

Find $\lim_{n \rightarrow \infty} A^n = ?$

2. (10%) Let

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Write $B = LU$, where L is a lower triangular matrix and U is an upper triangular matrix. Use this result to compute B^{-1} .

3. (10%) Test for all real vectors (x, y, z) , the function

$$f(x, y, z) = 5xy + 2xz - 5y^2 + 4yz - 3z^2$$

is positive definite, or not?

4. (10%) Find an orthogonal matrix that diagonalizes the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

5. (10%) For the matrix C , find a normalized QR decomposition for

$$C = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 3 & 4 & 2 & 6 \\ 2 & 5 & 1 & 4 \\ 2 & 0 & -1 & 4 \end{bmatrix}$$

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3. 試題隨卷繳回。(餘詳詳閱試場規則)

6. (10%) Find two solutions of

$$-x^2 y'' + 5x y' + 13y = 0$$

that are linearly independent for $x > 0$.

7. (10%) Solve the initial-value problem

$$y' + 2y = r(t), \quad y(0) = 4,$$

where

$$r(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 1, & 1 < t < \infty \end{cases}$$

is the ramp function.

8. (10%) Solve the initial-value problem

$$x'' + 4x' + 40x = 0, \quad x(0) = 3, \quad x'(0) = 12.$$

9. (10%) Find a particular solution of $y'(t) = By(t) + f(t)$, where

$$B = \begin{bmatrix} -4 & 2 \\ -3 & 3 \end{bmatrix}, \quad f(t) = \begin{bmatrix} 10 \\ 5t \end{bmatrix}$$

and $y(t)$ is a 2×1 vector.

10. (10%) Solve the initial-value problem

$$\frac{1}{8}x''(t) + 2x(t) = \frac{1}{2}\sin\frac{3\pi t}{2}, \quad x(0) = 0, \quad x'(0) = 0.$$

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