

國立暨南國際大學九十二學年度碩士班研究生入學考試試題

第 1 節 機率 適用: (資工所 412)

(本試題共 / 頁, 第 / 頁)

考生注意: 1. 依次序作答, 只要標明題號, 不必抄題。

2. 答案必須寫在答案卷上, 否則不予計分, 並限以藍黑色筆作答。

3. 試題隨卷繳回。(餘詳詳閱試場規則)

1. (12%) X_1 and X_2 are two independent Gaussian random variables. $E[X_1] = \mu_1$, $E[X_2] = \mu_2$, $\text{Var}[X_1] = \sigma_1^2 > 0$, $\text{Var}[X_2] = \sigma_2^2 > 0$. $Y = aX_1 + bX_2$, a, b are two non-zero constants. Prove that Y is also Gaussian.

2. (12%) X_1, X_2, \dots, X_n are n iid (independent and identically distributed) Laplace random variables (n is a positive integer) with the following PDF (probability density function).

$$f_X(x) = ae^{-2a|x|}, \quad -\infty < x < \infty, \quad a > 0$$

$Y = X_1 + X_2 + \dots + X_n$. Find the MGF (moment generating function) $\phi_Y(s) = E[e^{sY}]$ of Y .

3. (18%) Each pack of the cookie OREO contains a card. The color of the card can be equally likely to be red, yellow, or blue. The color of the card is independent from pack-to-pack. You decide to buy the cookies one pack after another until you obtain at least one of the cards for each of the three colors. N = the number of packs purchased. Find $E[N]$. (Hint: $N = N_1 + N_2 + N_3$, N_k = the number of packs purchased from $(k-1)$ colors to k colors, $k = 1, 2, 3$.)

4. (20%) X_1 and X_2 are two iid exponential random variables with the following PDF.

$$f_X(x) = \begin{cases} ae^{-ax} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad a > 0.$$

$Z = X_1 + X_2$. (a) Find the joint PDF $f_{X_1, Z}(x, z)$. (b) Find the PDF $f_Z(z)$ of Z .

5. (18%) X_1, X_2, X_3, X_4 are four iid Rayleigh random variables each with the following PDF $f_X(x)$.

$$f_X(x) = \begin{cases} axe^{-ax^2/2} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad a > 0.$$

$Y_1 = \min\{X_1, X_2\}$, $Y_2 = \min\{X_3, X_4\}$, and $Z = \max\{Y_1, Y_2\}$. Find the PDF $f_Z(z)$ of Z .

6. (20%) X and Y are two random variables with the following joint PDF

$$f_{X,Y}(x, y) = \begin{cases} 2(x^2 + y^2)/\pi & x^2 + y^2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$Z = -a \ln(1 - (X^2 + Y^2)^2)$, $a > 0$. Find the PDF $f_Z(z)$ of Z .