國立暨南國際大學九十三學年度轉學生入學考試試題　第二節

科目：微積分　適用：資工系二

編號：311

考生注意： 1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.試題隨卷繳回。

本試題
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第 ／ 頁

1. (10%) Find $\lim\limits_{n\to\infty}\dfrac{1}{n}\left\{\dfrac{1}{\sqrt{1^2+n^2}}+\dfrac{2}{\sqrt{2^2+n^2}}+\cdots\dfrac{n}{\sqrt{n^2+n^2}}\right\}$

   [Hint: limit of Riemann sums]

2. (25%)

   (a) (15%) Show that If f is continuous on [a,b] and c is any number in [a,b], then the function $F(x)=\int_c^x f(t)dt$ is continuous on [a,b], differentiable on (a,b), and satisfies $F'(x)=f(x)$ *for all x* in (a,b).

   (Hint: $\int_c^x f(t)dt=\int_c^a f(t)dt+\int_a^x f(t)dt$)

   (b) (10%) $\int_2^{x^3} f(t)dt=\sqrt{3x^3+1}-5$, find $f(x)$.

3. (15%) If the circle $x^2+(y-b)^2=a^2$, b>a>0, is rotated around the x-axis, the resulting "doughnut-shaped" solid is called a torus. Find the formula for the volume of the torus.

4. (20%)

   (a) (10%) Prove that $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

   (b) (10%) By (a), evaluate $\int_0^\pi \dfrac{x\sin x}{1+\cos^2 x}dx$.

5. (10%) The sequence $\{a_n\}$ defined by $a_n=\dfrac{n}{n+1}$

   (a) (2%) Show that it is increasing.

   (b) (3%) Find the greatest lower bound.

   (c) (5%) Find the least upper bound.

6. (10%) Show that

   (a) (5%) $\lim\limits_{x\to 0^+}\sqrt{x}\ln x=0$

   (b) (5%) $\lim\limits_{n\to\infty}(3^n+4^n)^{1/n}=4$

7. (10%) Determine the path of steepest descent along the surface $z=x^2+3y^2$ from the point $(1,1,4)$ to the point $(0,0,0)$.