國立暨南國際大學九十三學年度轉學生入學考試試題　　第 三 節

科目：微積分　適用：財金系二

編號：242

考生注意：　1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 試題隨卷繳回。

本 試 題
共 二 頁
第 一 頁

1. Consider the function $f(x) = \sqrt{2x-3}$ to answer the following questions.

   (i) Find the inverse function $f^{-1}(x)$ of $f(x)$. (3%)

   (ii) Write down the domain and the range of $f(x)$ and $f^{-1}(x)$ separately, and construct their relationship. (6%)

   (iii) Check your results in (i) algebraically by evaluating $f(f^{-1}(x))$ and $f^{-1}(f(x))$ both equal to x. (2%)

   (iv) Sketch the graphs of $f(x)$ and $f^{-1}(x)$. Are the graphs of $f(x)$ and $f^{-1}(x)$ reflections of each other? If they were, described the reflection line and prove it; if they were not, explained your results. (8%)

   (v) Do $f(x)$ and $f^{-1}(x)$ define themselves as functions? Are they one-to-one functions? How do you get your results? (6%)

2. Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false. (9%)

   (i) If $y = f(x)g(x)$, then $y' = f'(x)g'(x)$.

   (ii) If $f(x)$ is an nth-degree polynomial, then $f^{(n+1)}(x) = 0$.

   (iii) $\lim\limits_{x \to 1} \dfrac{e^x - 1}{\ln x} = \lim\limits_{x \to 1} \dfrac{e^x}{(1/x)} = \lim\limits_{x \to 1} xe^x = e$.

3. To find the derivative of the function. Simplify your result. (25%)

   (i) $h(x) = \left[x^2(2x+3)\right]^3$

   (ii) $h(t) = \dfrac{\sqrt{3t+1}}{(1-3t)^2}$

   (iii) $h(x) = \ln |\csc x^2 - \cot x^2|$

   (iv) $h(x) = \sec^{-1}\sqrt{x^2+1}$

   (v) $h(x) = 3x \cdot 3^x$

4. Evaluate the following integral. (30%)

   (i) $\displaystyle\int_0^4 \left(2 - |x-2|\right)dx$

   (ii) $\displaystyle\int \dfrac{1+e^{-x}}{1+xe^{-x}}dx$

   (iii) $\displaystyle\int x\sqrt{x^4 - 9}\,dx$

   (iv) $\displaystyle\int_0^2 \dfrac{x+2}{(x-1)^2}dx$

   (v) $\displaystyle\int \sec^3 x \, dx$

   (vi) $\displaystyle\int \dfrac{1}{2+\sin\phi+\cos\phi}d\phi$

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5. Sketch the graph of the function $f(x) = \dfrac{2x}{1+x^2}$. Label the intercept, relative extremes, points of inflection, and asymptotes. State the domain of the function. (5%)

6. Apply Taylor's Theorem to find the power series (centered at 0) for the function $f(x) = \dfrac{1}{x+1}$, and find the radius of convergence. (6%)