

科目：微積分 適用：土木系二

編號：321

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 試題隨卷繳回。

本試題
共 1 頁
第 1 頁

- Find the Taylor series of $f(x) = \ln(x)$ about $x=1$ and its interval of convergence. Also, use it to conjecture the value of $\lim_{x \rightarrow 1} \frac{\ln(x) - (x-1)}{(x-1)^2}$. (10 pts)
- Symbolically find δ in terms of ϵ for $\lim_{x \rightarrow 2} (x^2 + 1) = 5$. (10 pts)
- A curve described by the collection of endpoints of $\vec{r}(t) = \langle t, 4t-1, 2-6t \rangle$:
 - Find the arc length for $0 \leq t \leq 2$. (5 pts)
 - Find the curvature at $t=1$. (5 pts)
- $f(x, y) = x + 2y^2$, $R = \{(x, y) | 0 \leq x \leq 2, -1 \leq y \leq 1\}$
 - Express the Riemann sum for the given function f and region R with $n=3$, evaluating at midpoint. (5 pts)
 - Compute $\iint_R f(x, y) dA$. (5 pts)
- Use Jacobian between Cartesian coordinates and polar coordinates to derive the evaluation formula for polar coordinates: $\iint_R f(x, y) dx dy = \iint_S f(r \cos \theta, r \sin \theta) r dr d\theta$. Use the formula to evaluate $\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 2x dy dx$ by converting to polar coordinates. (10 pts)
- Determine if the series converges or diverges: (5 pts ea. 10 pts)
 - $\sum_{k=0}^{\infty} (-1)^k \frac{2k}{k+3}$ (hint: k^{th} -Term Test)
 - $\sum_{k=1}^{\infty} \frac{4^k}{(k!)^2}$ (hint: Ratio Test)
- Evaluate the Integral: (5 pts ea. 20 pts)
 - $\int_2^1 e^{\ln(x^2+1)} dx$
 - $\int x \csc x^2 \cdot \cot x^2 dx$
 - $\int x^3 \sin 2x dx$
 - $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$
- Find the derivative of y , (5 pts ea. 20 pts)
 - $y = \frac{1}{\sqrt{x}}$
 - $\sin(xy) + x^2 = x - y$
 - $y = \ln\left(\frac{x-2}{x+2}\right) \cos(2x^2)$
 - $y = \sqrt{\ln x + 1}$