國立暨南國際大學九十三學年度碩士班研究生入學考試試題

第 2 節離散數學　適用:(資管所 333　)

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. Explain the following terms: (a) poset; (b) non-planar graphs $K_{3,3}$ and K_5; (c) Pigeonhole Principle; (d) tautology; (e) group. (2 points for each)
2. The *Fibonacci numbers* F_n have the initial values $F_0 = 0$, $F_1 = 1$, and the recursion

   $F_n = F_{n-1} + F_{n-2}$ if $n \geq 2$. Use mathematical induction to prove the following identities.

   (a) $\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}$ if $n \geq 1$. (7 points)

   (b) $F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$ if $n \geq 0$. (8 points) *Hint*: $\left(\frac{1+\sqrt{5}}{2}\right)^2 = \frac{1+\sqrt{5}}{2} + 1$,

   $\left(\frac{1-\sqrt{5}}{2}\right)^2 = \frac{1-\sqrt{5}}{2} + 1$.
3. (a) Assume R is an equivalence relation on $X = \{3, 7, 8, 9, 13, 17, 32\}$ and aRb if $a \equiv b \pmod 4$, please draw the partition graph (equivalence classes) of R on X. (7 points)

   (b) Let (X, R) be a poset where X is the power set of $\{1, 2, 3\}$ and R is the inclusion relation on X, i.e., aRb if $a \subset b$, please draw the Hasse diagram of (X, R). (8 points)
4. Find the number of prime numbers that are less than or equal to 120 by the principle of inclusion and exclusion. (15 points)
5. There is a set of objects to be grouped into several containers. Some restrictions (such as chemical reaction) may exist between any pair of objects such that they cannot be put into the same container. We seek to distribute the objects using a minimum number of containers without violating the restrictions. Which of the following graph problems can be use to mathematically formulate the above problem? Why? (10 points) (1) Minimum spanning tree; (2) minimum vertex cover; (3) minimum graph coloring; (4) minimum cut.
6. (a) Find the negation of $\forall x(p(x) \Rightarrow \exists y(q(x, y) \wedge r(y)))$ (5 points)

   (b) We have the following four statements about logical equivalences. Identify the statements that are not logical equivalences. Give examples, say letting $p(x)$ denote "People x likes dogs." and $q(x)$ denote "People x likes cats.", for the statements that you have identified to explain why they are not logical equivalences. (10 points)

   (1) $\exists x(p(x) \wedge q(x)) \Leftrightarrow (\exists x p(x) \wedge \exists x q(x))$;　(2) $\exists x(p(x) \vee q(x)) \Leftrightarrow (\exists x p(x) \vee \exists x q(x))$;

   (3) $\forall x(p(x) \wedge q(x)) \Leftrightarrow (\forall x p(x) \wedge \forall x q(x))$;　(4) $\forall x(p(x) \vee q(x)) \Leftrightarrow (\forall x p(x) \vee \forall x q(x))$
7. (a) Show that for any undirected graph, the number of vertices of odd degrees is even. (5 points)

   (b) Let v_i be the vertex whose degree d_i is minimum in some graph. Now, we remove vertex v_i, its neighbor vertices and the edges on these vertices. Show that at least $d_i(d_i+1)/2$ edges have been removed. (5 points) If there are k neighbor vertices of v_i such that the k vertices are independent of one another (i.e. there is no edge between any two of the k vertices), show that the number of removed edges is at least $d_i(d_i+1)/2 + k(k-1)/2$. (5 points)
8. (a) Explain the definitions of Eulerian circuit and Hamiltonian path. (2 points)

   (b) Discuss the computational complexities required for finding them or checking whether they exist or not. (3 points)

END OF TEST