Engineering Mathematics

April 18, 2004

1. (30 pts.) Solve the following differential equations:

   (a) (15 pts.)

$$y'' - y' - 2y = 10\cos x,$$

   (b) (15 pts.)

$$\frac{d}{dt}x(t) = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} x(t) + 4\begin{bmatrix} e^{2t} \\ e^{4t} \end{bmatrix} \quad \text{with} \quad x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

2. (25 pts.) Two grounded, semi-infinite, parallel-plane electrodes are separated by a distance b. A third electrode perpendicular to and insulated from both is maintained at a constant potential V_o, as shown in Fig. 1. The potential distribution in the region enclosed by the electrodes can be determined by solving the famous 2-dimensional Laplace's equation:

$$\nabla^2 V(x,y) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

   using the technique of *Separation of Variables* subject to the following boundary conditions:

   - $V(x=0) = V_0, \quad V(x \to \infty) = 0,$
   - $V(y=0) = 0, \quad V(y=b) = 0.$

   Show the expression of $V(x,y)$.

3. (20 pts.) Find the current $i(t)$ in the circuit in Fig. 2 if a single square wave with voltage V_0 is applied. The circuit is assumed to be quiescent before the square wave is applied.

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(請詳閱試場規則)

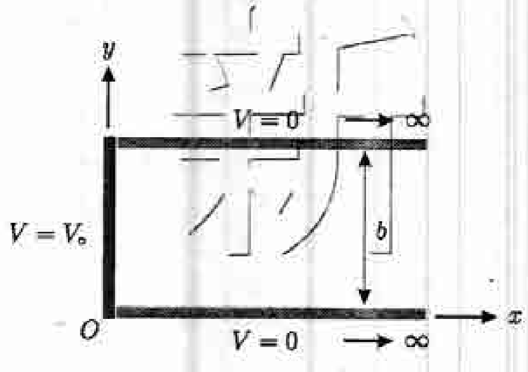

Figure 1: Figure for Problem 2.

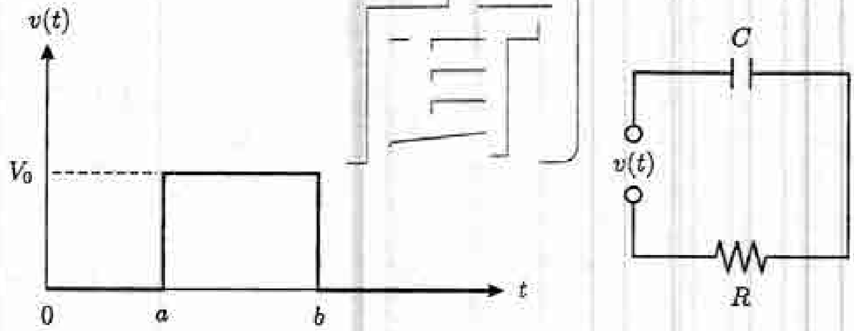

Figure 2: Figure for Problem 3.

4. (25 pts.) Consider the following continuous-time function

$$x(t) = \begin{cases} 1, & |t| \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(10 pts.)(a) Find the Fourier transform $X(j\omega)$ of the given function $x(t)$.

(15 pts.)(b) Assuming that $Y(j\omega) = X^2(j\omega)$, determine the inverse Fourier transform $y(t)$ of $Y(j\omega)$.