國立暨南國際大學九十三學年度碩士班研究生入學考試試題

第 2 節離散數學　適用：(資工所 413)　　(本試題共 / 頁，答 / 頁)

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回，(餘請詳閱試場規則)

1. Determine whether each of the following statements is true or false. For each false statement give a counterexample. (10 × 3%) **(答錯每小題倒扣 3%)**

   (a) If $a^2 \mid b^3$ then $a \mid b$.
   (b) If $(R, +, \cdot)$ is a ring with unity u_R, and S is a subring of R with unity u_S, then $u_R = u_S$.
   (c) Every finite field has a prime number of elements.
   (d) If a is a unit in a ring R, then a cannot be a proper divisor of zero.
   (e) If it is cool this Friday, then Craig will wear his suede jacket if the pockets are mended. The forecast for Friday calls for cool weather, but the pockets have not been mended. Therefore Craig won't be wearing his suede jacket this Friday.
   (f) $(A \cap B) \cup [B \cap ((C \cap D) \cup (C \cap \bar{D}))] = B \cap (A \cup C)$.
   (g) If $f: A \to B$ is one-to-one and $g, h: B \to C$ with $g \circ f = h \circ f$, then $g = h$.
   (h) Let $\Sigma = \{w, x, y\}$. If $A_1 = \{x^i y^j \mid i, j \in \mathbf{Z}^+, j > i \geq 1\}$, $A_2 = \{w^i y^j \mid i, j \in \mathbf{Z}^+, i > j \geq 1\}$, then $A_1 \cup A_2$ is a language over Σ.
   (i) Let A be a set and I an index set where, for each $i \in I$, $\mathcal{R}_i$ is a relation on A. $\bigcap_{i \in I} \mathcal{R}_i$ is reflexive on A if and only if each $\mathcal{R}_i$ is reflexive on A.
   (j) The union of the edge sets of distinct u, v-walks must contain a cycle.

(以下各題均須寫出計算過程方予計分)

2. In the following program segment, i, j, k, l, m and *counter* are integer variables. Determine the value that the variable *counter* will have after segment is executed. (10%)

```
counter := 105
for i := 1 to 10 do
  for j := i to 10 do
    for k := j to 10 do
      for l := k to 10 do
        for m := l to 10 do
          counter := counter + 1
```

3. Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

   (a) Compute A^2, A^3, and A^4. (3%)
   (b) Conjecture a general formula for A^n, $n \in \mathbf{Z}^+$, and establish your conjecture by mathematical induction. (17%)

4. A pair of dice, one red and the other green, is rolled six times. We know that the ordered pairs (1, 1), (1, 5), (2, 4), (3, 6), (4, 2), (4, 4), (5, 1), and (6, 3) did not come up. What is the probability that every value came up on both the red die and the green one? (15 %)

5. If $G = (V, E)$ is a loop-free undirected graph, we call G *color-critical* if $\chi(G - v) < \chi(G)$ for all $v \in V$. A vertex v in G is called an *articulation point* if $\kappa(G - v) > \kappa(G)$; that is, the subgraph $G - v$ has more components than the given graph G. Prove that a color-critical graph has no articulation points. (15%)

6. Let $(G, \circ)$ be a group where $x \circ a \circ y = b \circ a \circ c \Rightarrow x \circ y = b \circ c$, for all $a, b, c, x, y \in G$. Prove that $(G, \circ)$ is an abelian group. (10%)