考生注意: 1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘請詳閱試場規則)

1. Let $y_t = \phi y_{t-1} + \epsilon_t$, where $|\phi| < 1$, ϵ_t IID $N(0, \sigma^2)$, $y_0 = 0$.

   (a) Let $\gamma(k)$ be the k^{th} autocovariance of y_t. Find $\gamma(k)$. (10 points)

   (b) What is the log-likelihood function of $y_1, y_2, \cdots, y_n$? (10 points)

2. Suppose $y_t = \phi y_{t-1} + u_t$, $u_t = \rho u_{t-1} + e_t$, where $|\phi| < 1$, $\rho \neq 0$.

   (a) Explain why $E(y_{t-1}u_t) \neq 0$. (10 points)

   (b) Let $\tilde{\phi}$ be the LS estimator of ϕ. Is $\tilde{\phi}$ consistent for ϕ? Explain your answer. (10 points)

   (c) How do you test whether u_t is serially correlated or not? (10 points)

3. Suppose that 40 percent of the students in a large population are freshmen, 30 percent are sophomores, 20 percent are juniors, and 10 percent are seniors. Suppose that 10 students are selected at random from the population; and let X_1, X_2, X_3, X_4 denote, respectively, the numbers of freshmen, sophomores, juniors and seniors that are obtained.

   (a) Determine $\rho(X_i, X_j)$ for each pair of values i and j $(i < j)$. (5%)

   (b) For what values of i and j $(i < j)$ is $\rho(X_i, X_j)$ must negative? (5%)

   (c) For what values of i and j $(i < j)$ is $\rho(X_i, X_j)$ close to zero? (5%)

4. Suppose that $X_1, \cdots, X_n$ form a random sample from a distribution for which the p. d. f. $f(x|\theta)$ is as follows:

$$f(x|\theta) = \begin{cases} \theta x^{\theta-1} & \text{for } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

   Also suppose that the value of θ is unknown $(0 < \theta)$.

   (a) Find the maximum likelihood estimator (M.L.E.) of θ. (7%)

   (b) Determine the asymptotic distribution of the M.L.E. of θ. (8%)

5. Suppose that a certain drug A was administered to eight patients at random; and after a fixed time period, the concentration of the drug in certain body cells of each patient was measured in appropriate units. Suppose that these concentrations for the eight patients were found to be as follows:

1.23, 1.42, 1.41, 1.62, 1.55, 1.51, 1.60, and 1.76

Suppose also that a second drug B was administered to six different patients selected at random; and that when the concentration of drug B was measured in a similar way for six patients, the results were as follows:

1.76, 1.41, 1.87, 1.49, 1.67, and 1.81.

Let μ_A denote the mean of each observation for drug A, and let μ_B denote the mean of each observation for drug B. Also suppose that all observations have a common unknown variance. Please find a confidence interval for $\mu_A - \mu_B$ with confidence coefficient 0.90 under the following hypotheses:

$H_0: \mu_A - \mu_B = 0$; $H_1: \mu_A - \mu_B \neq 0$. (10%)

6. Consider a problem of simple linear regression of Y on X and that observations $Y_1,\ldots,Y_n$ are independent. Let $Z_i = Y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i$ denote the residual of the observation Y_i $(i=1,\ldots,n)$. Evaluate $Var(Z_i)$ for given values of $x_1,\ldots,x_n$, and show that it is a decrease function of the distance between x_i and $\bar{x}_n$. (10%)