科目：**411** 離散數學
系組：資工系

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 試題隨卷繳回。

The following problems may be answered in Chinese or English. You need to give all details in order to receive any credit (point).

Let A be a nonempty set.

Definition 1 *A function $f : A \longrightarrow A$ is compatible with a binary relation $R \subseteq A \times A$ if and only if for all x and y in A*

$$xRy \Longrightarrow f(x)Rf(y).$$

Definition 2 *Let $R_1 \subseteq A \times A$ be an equivalent relation and $g : A \Longrightarrow A$. Let $R_2 \subseteq (A/R_1) \times (A/R_1)$ be defined as follows:*

$$[x]_{R_1} R_2 [y]_{R_1} \Longleftrightarrow [y]_{R_1} = [g(x)]_{R_1},$$

where A/R is the partition of A induced by an equivalent relation R and $[x]_R$ denotes the equivalent class of x with respect to R.

1. (20 points) Let $A = N$, $k \in I_+$ and $aRb \Longleftrightarrow a \equiv b \ (mod\ k)$. Show that if $f(a) = a^2, \forall a \in N$, then f is compatible with R, where N is the set of nonnegative integers and I_+ is the set of positive integers.

2. (10 points) Let $A = \{a, b, c\}$, $R_1 = \{< a, a >, < a, b >, < b, a >, < b, b >, < c, c >\}$ and $g = \{< a, c >, < b, a >, < c, a >\}$. Construct R_2 as defined in Definition 2. Is R_2 a function? Prove your answer.

3. (10 points) Let $A = \{a, b, c\}$, $R_1 = \{< a, a >, < a, b >, < b, a >, < b, b >, < c, c >\}$ and $g = \{< a, b >, < b, b >, < c, c >\}$. Construct R_2 as defined in Definition 2. Is R_2 a function? Prove your answer.

4. (30 points) Prove that if g is not compatible with R_1, then R_2 is not a function, where g, R_1, R_2 are as given in Definition 2.

5. (30 points) Prove that if g is compatible with R_1, then R_2 is a function, where g, R_1, R_2 are as given in Definition 2.