科目：312 統計學

系組：國企系甲組

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 試題隨卷繳回。

Useful information: For $Z \sim N(0,1)$, $\Pr(Z > 2.57) = 0.005$, $\Pr(Z > 2.32) = 0.01$, $\Pr(Z > 1.64) = 0.05$, $\Pr(Z > 1.28) = 0.10$.

1. (20 points) Suppose $\overline{X}$ is the mean of 100 observations from a population with mean μ and variance $\sigma^2 = 25$. Find limits between which $\overline{X} - \mu$ will lie with probability at least .90. Use both Chebychev's Inequality and the Central Limit Theorem, and comment on each other.

2. (20 points) Two movie theaters compete for the business of 2,000 customers. Assume that each customers choose between the movie theaters independently and with "difference." Let N denote the number of seats in each theater.

   (a) Using a binomial model, find an expression for N that will guarantee that the probability of turning away a customer (because a full house) is less than 1%.

   (b) Use the normal approximation to get a numerical value for N.

3. (20 points) Suppose the random variable X has a lognormal distribution.

   (a) Show that the pdf of X is

   $$f(x|\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma}\frac{1}{x}\, e^{-(\log x-\mu)^2/(2\sigma^2)}, \quad 0 < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

   (b) Calculate $E(X)$ and $Var(X)$.

4. (20 points) Let $X_1, \ldots, X_n$ be a random sample, where $\overline{X} = \frac{1}{n}\sum_{i=1}^n X_i$ and $S^2 = \frac{1}{n-1}\sum_{i=1}^n (X_i - \overline{X})^2$.

   (a) Show that

   $$S^2 = \frac{1}{2n(n-1)}\sum_{i=1}^n\sum_{j=1}^n (X_i - X_j)^2$$

   (b) Assume that $X_1, \ldots, X_n$ have a finite fourth moment, and denote $\theta_1 = EX_i$, $\theta_j = E(X_i - \theta_1)^j$, $j = 2, 3, 4$. Show that $Var(S^2) = \frac{1}{n}(\theta_4 - \frac{n-3}{n-1}\theta_2^2)$.

5. (20 points) Let $X_1, \ldots, X_n$ be iid with pdf

   $$f(x|\theta) = \theta^x(1-\theta)^{(1-x)}, \quad x = 0 \text{ or } 1, 0 \le \theta \le \frac{1}{2}$$

   (a) Find the method of moments estimator and MLE of θ.

   (b) Find the mean squared errors of each of the estimators.

   (c) Which estimator is preferred? Justify your answer.