科目：453 流體力學
系組：土木系

考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 試題隨卷繳回。

1. (30%)

The equations governing steady, two dimensional, plane flow of an inviscid, incompressible fluid, in which the gravity may be neglected, are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1.1}$$

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x} \tag{1.2}$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y} \tag{1.3}$$

If the fluid is stratified, the density ρ will depend on both x and y.

Show that the transformation,

$$u^*=\sqrt{\frac{\rho}{\rho_0}}u,\quad v^*=\sqrt{\frac{\rho}{\rho_0}}v,$$

where ρ_0 is a constant reference density, transforms Eqs.(1.1) to (1.3) into those of a constant-density fluid whose velocity components are u^* and v^*.

2. Given an unsteady, two dimensional velocity field as (20%)

$$u=x(1+2t),\quad v=y. \tag{2.1}$$

Answer the following questions:

(1) Does the velocity field (2.1) represent an incompressible fluid flow ? (5%)

(2) Is the flow irrotational ? (5%)

(3) Find the streamline which passes through the point (1,1) at time $t=0$. (5%)

(4) Find the pathline which passes through the point (1,1) at time $t=0$. (5%)

3. (20%) The thermal energy equation is given by

$$\rho\frac{De}{Dt}=-p\nabla\bullet\mathbf{u}+\nabla\bullet(k\nabla T)+\Phi, \tag{3.1}$$

where $\mathbf{u}$ is the velocity vector, ρ the density, e the internal energy, p the pressure, k the thermal conductivity, T the temperature, Φ the dissipation function.

By using the definition of the enthalpy $h=e+p/\rho$, show that an equivalent form of equation (3.1) is given by

$$\rho\frac{Dh}{Dt}=\frac{Dp}{Dt}+\nabla\bullet(k\nabla T)+\Phi \tag{3.2}$$

4. (30%) A steady, two dimensional viscous fluid flow driven by a stretching sheet $y=0$, with the speed $u=cx, c>0$; therefore the fluid is occupied

科目：453 流體力學
系組：土木系

考生注意：
1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 試題隨卷繳回。

above the sheet $y>0$. Using the boundary layer approximation we have the governing equations, neglecting the pressure gradient and gravity effects, of the form

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{4.1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}, \tag{4.2}$$

The appropriate boundary conditions are

$$u=cx,\ v=0 \ \text{ at } \ y=0, \tag{4.3}$$

$$u\to 0 \ \text{ as } \ y\to\infty. \tag{4.4}$$

Using the similarity transformation

$$u=cxf'(\eta),\ v=-\sqrt{\frac{c\mu}{\rho}}f(\eta) \ \text{ and } \ \eta=\sqrt{\frac{c\rho}{\mu}}y, \tag{4.5}$$

where the prime denotes differentiation with respect to η.

(1) Show that the continuity equation (4.1) has been satisfied by (4.5). (7%)

(2) Show that the governing equation (4.2) can be transformed intto (7%)

$$f'^2-ff''=f''', \tag{4.6}$$

(3) Show that the transformed boundary conditions are (8%)

$$f=0,\ f'=1 \ \text{ at } \ \eta=0, \tag{4.7}$$

$$f'\to 0 \ \text{ at } \ \eta\to\infty. \tag{4.8}$$

(4) Show that the appropriate solution to (4.6) subject to (4.7) and (4.8) is of the form (8%)

$$f=1-e^{-\eta}$$