

國立暨南國際大學九十三年學年度碩士班研究生入學考試試題

第 1 節 機率 適用：(資工所 412)

(本試題共 / 頁，第 / 頁)

- 考生注意：1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分，並限以藍黑色筆作答。
3. 試題隨卷繳回。(餘詳閱試場規則)

1. (18%) X and Y are two continuous random variables with the following joint PDF(probability density function) $f_{X,Y}(x,y)$.

$$f_{X,Y}(x,y) = \begin{cases} 4e^{-2y} & 0 \leq x < y \\ 0 & \text{otherwise} \end{cases}$$

- (a) Given $X = x \geq 0$, find the conditional PDF $f_{Y|X}(y|x)$ of Y . (b) Find the mean $E[Y]$ and the variance $\text{Var}[Y]$ of Y .

2. (14%) X_1 and X_2 are two joint Gaussian random variables. $E[X_1] = E[X_2] = 0$. $\text{Var}[X_1] = 10$. $\text{Var}[X_2] = 5$. $E[X_1 X_2] = 1$.

Let $Y_1 = (2X_1 + X_2)$ and $Y_2 = (X_1 - 3X_2)$. Find the joint PDF $f_{Y_1, Y_2}(y_1, y_2)$ of Y_1 and Y_2 .

3. (18%) $X_1, X_2, X_3, X_4, X_5, X_6$ are six iid(independent and identically distributed) continuous random variables each with the following PDF $f_X(x)$.

$$f_X(x) = \begin{cases} 2x & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y_1 = \max\{X_1, X_2, X_3\}$, $Y_2 = \max\{X_4, X_5, X_6\}$, and $Z = \min\{Y_1, Y_2\}$. Find the PDF $f_Z(z)$ of Z .

4. (18%) X and Y are two random variables with the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 1 & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Let event $A = \{(X+Y) \leq 1\}$, and $Z = (X+Y)^2$. Find the conditional PDF $f_{Z|A}(z)$.

5. (18%) X and Y are two independent Gaussian random variables. $E[X] = E[Y] = 0$. $\text{Var}[X] = \text{Var}[Y] = 1$. Let $R = \sqrt{X^2 + Y^2}$, and W = the phase of $(\frac{Y}{X})$, i.e.,

$$W = \begin{cases} \tan^{-1}(\frac{Y}{X}) & X > 0 \\ \tan^{-1}(\frac{Y}{X}) - \pi & X < 0, Y < 0 \\ \tan^{-1}(\frac{Y}{X}) + \pi & X < 0, Y > 0 \end{cases}$$

Therefore, $X = R \cos(W)$ and $Y = R \sin(W)$. Prove that R and W are independent.

6. (14%) X and Y are two independent Poisson random variables and $E[X] = 8$, $E[Y] = 10$. Let $Z = (X+Y)$. Find the MGF(moment generating function) $\phi_Z(s) = E[e^{sZ}]$ of Z .