國立暨南國際大學九十四學年度轉學生入學考試試題

第 3 節

科目：微積分　適用：資工系二

編號：312

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

1) (10 pts) Prove the following limits by using the definition of limit of a sequence.

(a)(5 pts) $\lim_{n\to\infty} \frac{3n+8}{2n+9} = \frac{3}{2}$

(b)(5 pts) $\lim_{n\to\infty} \frac{(-1)^n n}{n^2+1} = 0$

2) (15 pts)

(a)(7 pts) Calculate the derivative

$$\frac{d}{dx}\left(\int_{\sqrt{x}}^{x^2+x} \frac{dt}{2+\sqrt{t}}\right).$$

(b)(8 pts) Let f be continuous, Calculate the derivative

$$\frac{d}{dx}\left(\int_0^x \left[t\int_1^t f(u)du\right] dt\right).$$

3) (20 pts) Define $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} x^2\left[\sin\frac{1}{x} - 2\right], & \text{for } x \neq 0, \\ 0, & \text{for } x = 0 \end{cases},$$

(a)(5 pts) Prove that $f(x)$ has a strict maximum at $x = 0$ (i.e. $f(0) > f(x)$ for all $x \neq 0$).

(b)(5 pts) Prove that $f(x)$ is differentiable on $\mathbb{R}$

(c)(10 pts) Prove that $f(x)$ is not increasing on the interval $(-\epsilon, 0)$ and $f(x)$ is not decreasing on the interval $(0, \epsilon)$ for any $\epsilon > 0$.

4) (15 pts) Let $f(x) = \frac{\ln x}{x}$ on $[1, 2e]$

(a)(5 pts) Find the area bounded by $f(x)$ and the x-axis.

(b)(10 pts) Find the volume of the solid generated by revolving the region in part (a) around the x-axis.

5) (15 pts) Let $a > 0$. Give a definition of the following improper Riemann integral as a limit of Riemann integrals:

$$\int_2^\infty \frac{1}{x(\log x)^a} dx.$$

For what values of a does this integral converge.

6) (15 pts) Let $x \in \mathbb{R}$, find the interval of convergence of the following power series:

$$\sum_{n=0}^\infty \frac{1}{1+n2^n} x^n.$$

7) (10 pts) Given that $0 < a < b$, find the absolute maximum value taken on by the function

$$f(x,y) = \frac{xy}{(a+x)(x+y)(b+y)}$$

on the open square $\{(x,y) : a < x < b, a < y < b\}$.