

科目：離散數學 適用：資工所

編號：413

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題  
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第一頁

1. Determine whether each of the following statements is true or false. For each false statement give a counterexample. (10 × 3%) (答錯每小題倒扣3%)

- (a)  $(A \cap B) \cup (B \cap C) \supseteq A \cup B$ .
- (b) If  $x \in \Sigma^*$  and  $\|x^3\| = 27$ , then  $\|x\| = 3$ .
- (c)  $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}$  and  $\exists z \in \mathbb{Z}, (x = 7y + 3z)$ .
- (d) For any finite field  $F$ , the order of  $F$  is prime.
- (e) For all positive integers  $m$  and  $n$ ,  $n \binom{m+n}{m} = (m+1) \binom{m+n}{m+1}$ .
- (f) Let  $f: A \rightarrow B$ , with  $A_1, A_2 \subseteq A$ . Then  $f(A_1 \cap A_2) = f(A_1) \cap f(A_2)$ .
- (g) Let  $F_n$  denote the  $n$ th Fibonacci number, then  $F_n \leq (5/3)^n$  for all  $n \in \mathbb{N}$ .
- (h) Let  $n$  be a fixed positive integer that satisfies the property: For all  $a, b \in \mathbb{Z}^+$ , if  $n|ab$  then  $n|a$  or  $n|b$ . Then  $n = 1$  or  $n$  is prime.
- (i) If  $\mathcal{R}$  is a relation on a set  $A$ , and  $\mathcal{R}$  is both an equivalence relation and a partial order on  $A$ , then  $\mathcal{R}$  must be the equality relation on  $A$ .
- (j) If Ron's computer program is correct, then he'll be able to complete his computer science assignment in at most three hours. It takes Ron two hours to complete his computer science assignment. Therefore Ron's computer program is correct.

(以下各題均須寫出計算過程方予計分)

2. (10%) For every tree  $T = (V, E)$ , if  $|V| \geq 2$ , then  $T$  has at least two pendant vertices — that is, vertices of degree 1.
3. (10%) If  $|A| = 30$  and the equivalence relation  $\mathcal{R}$  on  $A$  partitions  $A$  into (disjoint) equivalence classes  $A_1, A_2$  and  $A_3$ , where  $|A_1| = |A_2| = |A_3|$ , what is  $|\mathcal{R}|$ ?
4. (10%) If a fair die is rolled 12 times, what is the probability that the sum of the rolls is 30? (You can write the answer in the following form:  $\binom{18}{15} + 4\binom{17}{14} + 6\binom{16}{13} + 4\binom{15}{12} + \binom{14}{11}$ , and need not evaluate it to the simplest answer.)
5. (15%) If  $G$  is a finite group of order  $n$  with  $H$  a subgroup of order  $m$ , then  $m$  divides  $n$ .
6. (10%) In how many ways can four w's, four x's, four y's, and four z's be arranged so that there is no consecutive quadruple of the same letter?
7. (15%) Solve the system of recurrence relation.

$$a_{n+1} = -2a_n - 4b_n$$

$$b_{n+1} = 4a_n + 6b_n$$

$$n \geq 0, a_0 = 1, b_0 = 0.$$