

科目：統計學 適用：資管所

編號：342

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. The Sales Report of NCNU Supermarket indicates that 90 boxes of dumplings of three different flavors were sold in last month. Among these 90 boxes, 31 boxes are of the flavor of Pork&Cabbage, 39 boxes of Pork&Leek, 20 boxes of Vegetarian, respectively. Do the people in NCNU have a preference for one of the three flavors of dumplings? (10 points)
2. Customers arrive at a certain website according to an approximate Poisson process at a mean rate of 20 per hour. Assume that the processing time for each customer is very small and can thus be neglected. What is the probability that the web server can stay idle (i.e., no customers arrive) for more than 5 minutes? (10 points)
3. Consider the relation between two variables x and Y with a given data set of $\{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$. We have the following linear model to predict the future observed value of Y based on a known variable x .

$$Y_i = \alpha + \beta(x_i - \bar{x}) + \xi_i,$$

where $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$ and ξ_i , for $i=1, 2, \dots, n$, are independent and normally

distributed with $N(0, \sigma^2)$.

- (1) Please find the least-squares estimators of α and β . (10 points)
- (2) Given $E[Y|x=1]=0.5$ and $E[Y|x=5]=4.7$, please find the linear regression line? (5 points)
- (3) Given the standard deviations $S_x=1.2$ and $S_y=1.5$, what is the correlation coefficient of x and Y ? (5 points)
4. (1) What is a Type I error? Please give an example. (10 points)
- (2) What is a type II error? Please give an example. (10 points)
5. If Y is the number of successes in n Bernoulli trials with probability p of success on each trial, for $\xi > 0$, please prove

$$P\left(\left|\frac{Y}{n} - p\right| \geq \xi\right) \leq \frac{p(1-p)}{n\xi^2}. \quad (10 \text{ points})$$

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6. An experiment was performed to compare the efficiencies of two different designs of fans. Twelve pieces of fans of design 1 and ten pieces of fans of design 2 were randomly selected and tested, respectively. The samples of design 1 gave an average efficiency of 85% with a sample standard deviation of 4, while the samples of design 2 gave an average of 81% and a sample standard deviation of 5. Can we conclude at the 0.05 level of significance that the efficiency of fans of design 1 exceeds that of design 2 by more than 2%? (10 points)

7. Consider the distribution function of Z

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw.$$

Some of the values of $\Phi(z)$ are listed as below:

z	-12.5	-0.75	-0.5	0.2	0.5	1.25
$\Phi(z)$	0.1056	0.2266	0.3085	0.5987	0.6915	0.8944

Let random variable X have the normal distribution $N(3, 16)$. Please find

(1) $P(4 < X \leq 8) = ?$ (5 points)

(2) $P(-2 \leq X \leq 1) = ?$ (5 points)

8. Suppose that a city of $N=100,000$ citizens is interested in assessing the support for a new policy of waste recycling. To estimate the proportion p in favor of the new policy, how large a sample is required so that with 95% confidence the maximum error of the estimate of p is 0.03? (10 points)

Remarks:

$$F_{0.025}(10, 12) = 3.37$$

$$t_{0.05}(20) = 1.725$$

$$t_{0.025}(20) = 2.086$$

$$z_{0.05} = 1.64$$

$$\chi^2_{0.05}(2) = 5.991$$

$$F_{0.975}(10, 12) = 0.276$$

$$t_{0.05}(22) = 2.819$$

$$t_{0.025}(22) = 2.074$$

$$z_{0.025} = 1.96$$

$$\chi^2_{0.025}(2) = 7.378$$