科目：通訊系統導論 適用：通訊所

編號：452

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

1) (10 pts) Explain the functions of the preemphasis and deemphasis filters used in a frequency modulation (FM) system.

2) (10 pts) Describe the central-limit theorem (CLT) and Chebyshev's inequality in your probability class.

3) (10 pts) Explain the sampling theorem for lowpass signals of bandwidth W.

4) (10 pts) Describe what the Wiener-Khinchine theorem states regarding an auto-correlation function.

5) (10 pts) Explain Nyquist's pulse-shaping criterion for a band-limited channel with intersymbol-interference (ISI).

6) (20 pts) Consider the binary erasure channel, which is shown in the Fig. 1. This binary

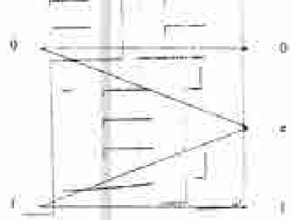

Fig. 1. Binary erasure channel

erasure channel has two inputs $s = 0, 1$ and three outputs $d = 0, 1, e$. Moreover, assume that the inputs are equally likely. Now suppose

$$p_{Y|s}(y|s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(y-(-1)^{s+1})^2}{2\sigma^2}\right], s = 0, 1, -\infty < y < \infty.$$

That is, the received random variable, Y, has Gaussian distribution $N(-1, \sigma^2)$ when the input is 0, and Y has Gaussian distribution $N(1, \sigma^2)$ when the input is 1. Assume that the cost is given by

$$C(d, s) = \begin{cases} 1, & \text{if } d = 1, s = 0 \text{ or } d = 0, s = 1; \\ 0, & \text{if } d = 0, s = 0 \text{ or } d = 1, s = 1; \\ c, & \text{if } d = e. \end{cases}$$

Given $Y = y$, the optimal Bayes decision rule chooses the decision with smallest *a posteriori* cost given by

$$C(d|y) = \sum_{j=0}^{1} C(d, j) P(s = j|y).$$

That is, choose $d = \hat{d}$ if $C(\hat{d}|y) = \min\{C(0|y), C(1|y), C(e|y)\}$.

(a) (10 pts) Find the optimal Bayes decision rule if $c \geq 0.5$.

(b) (10 pts) Find the optimal Bayes decision rule if $c < 0.5$.

7) (15 pts) We consider the problem of estimating a parameter Θ observed in additive noise. The observations are given by

$$Z_i = \Theta + V_i, i = 1, \ldots, N$$

We assume that the V_i are independent, identically distributed Gaussian random variables with zero mean and variance σ_V^2. We also assume that Θ is Gaussian, zero mean, and with variance σ_θ^2. What is the MAP (maximum *a posteriori* probability) estimate $\hat{\theta}_{MAP}$ for θ?

8) The probability density function (pdf) of a Gaussian random variable X is given by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Consider another random variable defined as

$$Y = e^X$$

(a) (5 pts) Find the pdf for Y.

(b) (5 pts) Find the mean, median, and variance of Y. Examine the mean and median for Y and find out which one is larger.

(c) (5 pts) Define the complementary cumulative distribution function for zero mean and unit variance Gaussian random variable as

$$Q(a) = \frac{1}{\sqrt{2\pi}} \int_a^\infty e^{-t^2/2} dt$$

Express $P[Y > y]$ using $Q(\cdot)$ function for $y > 0$.