九十四學年度經濟所碩士班統計學入學考題

（禁止使用計算機）

1. Let the joint probability density function of random variables, X and Y follow $f_{X,Y}(a,b) = c(a+b)$, where a=0,1,2; b=1,2,3 and c is one unknown real number.
   (1) What is the value of $\mathrm{cov}(X,Y)$? (5%)
   (2) What is the value of $IE(3+0.2Y|X=2)$? (5%)

2. Let $X_i \sim N(\mu_i, \sigma_i^2), i = 1,2,3,.....n$ and X_i could not be independent each other. Does any linear combination among $X_i, i = 1,.....,n$ still follow one normal distribution? Please explain your answer. (10%)

3. We often utilize the unbiasedness, efficiency and consistency to evaluate the characteristics of one point estimator. Can we say that an unbiased estimator is consistent or one consistent estimator is unbiased? You could explain it with examples. (10%)

4. One teacher randomly chooses two students, Willy and Julia. Their respective course credits as the following Table one:

Table one

	Math	English	History
Willy	91	76	82
Julia	75	90	84

   (1) Is Willy's performance different from what of Julia ? (5%)
   (2) What is the cross effect between gender and course? (5%)

5. The following table reveals the data of traveling abroad of fifty people.

times of traveling	real number of traveling people	theoretical number of traveling people
1	32	30.95
2	12	14.85
3	6	4.2
>3	0	0

Does the number of traveling abroad follow one Poisson distribution? (10%)

($x^2_{(1,0.05)} = 3.841$)

6. Suppose the linear regression model is given by $y_i = \beta x_i + u_i$, where x_i are positive and nonstochastic, and the disturbance u_i are independent with $E(u_i) = 0$ and $V(u_i) = \sigma^2 x_i$ for all $i = 1,2,\cdots,n$.

(1) Derive the least squares estimator $\hat{\beta}$ for β. (10 %)

(2) Derive the weighted least squares estimator $\hat{\beta}_w$ for β. (10 %)

(3) Explain why $V(\hat{\beta}_w)$ is smaller than $V(\hat{\beta})$. (10 %)

7. Suppose the linear regression model is given by $y_i = \alpha + \beta x_i + u_i$ for $i = 1,2,\cdots,n$.

(1) Write down the conditions you need to ensure that the least squares estimator $\hat{\beta}$ is unbiased for β. (10 %)

(2) Consider another estimator $\tilde{\beta} = (y_2 - y_1)/(x_2 - x_1)$ for β. Under the same conditions in (1), is $\tilde{\beta}$ unbiased for β? (10 %)