

科目：微積分 適用：土木系二

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

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Note: Numerical calculations are NOT required BUT all answers must be in the simplest form.

1. Evaluate the Integrals: (5 pts ea. 20 pts)

(a) $\int \frac{1}{3\sin x - 4\cos x} dx$ (hint: use $t = \tan(x/2)$ substitution)

(b) $\int_0^1 e^t \sin(t-s) ds$ (hint: use integration by parts)

(c) $\int \frac{\ln(x+1)}{x^2} dx$

(d) $\int \frac{dx}{x^2 - a^2}$

2. Find the first derivatives of y and simplify it. (5 pts ea. 25 pts)

(a) $\tan(x/y) = x + y$ (hint: implicit derivative)

(b) $y = \cos^n x \cos nx$

(c) $y = x^{\sin x}$

(d) $y = \tan^{-1} \sqrt{x}$

(e) $y = e^{\cosh 3x}$

3. Find the velocity and acceleration vectors for the position vector $\vec{r}(t) = \langle 4\sin t, 4\cos 2t, 4e^t \rangle$ at $t = \pi/2$. (10 pts)4. Symbolically find δ in terms of ϵ for $\lim_{x \rightarrow 2} (x^2 - 1) = 3$. (10 pts)5. Find the Maclaurin series of $f(x) = \tan^{-1} x$ and its interval of convergence. Also, use it todetermine the value of $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x}$. (10 pts)6. Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 4$ and theplane $y + z = 2$ with parameter t . Also, find the arc length of the intersection curve for $0 \leq t \leq \pi$. (10 pts)7. The Bessel function of order 0 is defined by $J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2}$, answer the following questions:(a) Find the domain of $J_0(x)$ by the Ratio Test. (5 pts)(b) Show that $J_0(x)$ satisfies the differential equation: $x^2 J_0''(x) + x J_0'(x) + x^2 J_0(x) = 0$, where $J_0''(x)$ and $J_0'(x)$ are the second and first derivative of $J_0(x)$, respectively. (5 pts)(c) Evaluate the first 3 terms of $\int_0^1 J_0(x) dx$ without calculating the number. (5 pts)