

科目：工程數學 適用：通訊所

編號：451

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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- (1) (10 %) State the Central Limit Theorem and prove it.
 - (2) (5 %) Given n independent positive random variables x_1, \dots, x_n , we form their product: $y = x_1 x_2 \cdots x_n, x_i > 0$. Show that for large n , the density of y is approximately lognormal.
- Given a discrete type random variable n taking the values $1, 2, \dots$ and a sequence of random variables x_k independent of n , we form the sum:
$$s = \sum_{k=1}^n x_k.$$
 - (1) (5 %) If the random variables x_k have the same mean η , derive the expression for $E\{s\}$.
 - (2) (10 %) If the random variables x_k are uncorrelated with the same variance σ^2 , derive the expression for $E\{s^2\}$.
- (1) (10 %) Solve $x'' + 3x' + 2x = \delta(t - \pi) + \delta(t - 2\pi), x(0) = 0, x'(0) = 0$.
 - (2) (10 %) Solve $\frac{dr}{dt} = \frac{1}{\sin t - r \tan t}, 0 < t < \pi/2$.
- (1) (10 %) Solve $x^2 y'' + x y' - y = 0, x > 0$.
 - (2) (10 %) Solve $x^2 y'' - 3 y' + 4y = 0, x > 0$.
- (1) (3 %) Find the solution to the system
$$\begin{aligned} u + v + w &= 0 \\ u + 2v + 3w &= 0 \\ 2u + 5v + 7w &= 1 \end{aligned}$$
 - (2) (2 %) Determine whether there is a solution to the system
$$\begin{aligned} u + v + w &= 0 \\ u + 2v + 3w &= 0 \\ 3u + 5v + 7w &= 1 \end{aligned}$$
 - (3) (10 %) Find the least square solution to the system
$$\begin{aligned} u + 2v - w &= 4 \\ u + v + 4w &= 8 \\ u - 2v + 2w &= 3 \\ u - 2v - 3w &= -2 \end{aligned}$$
- (15 %) Let u_1 and u_2 form an orthonormal basis for R^2 and let u be a unit vector in R^2 . If $u^T u_1 = \frac{1}{2}$, determine the value of $|u^T u_2|$.