

科目：機率 適用：資工所

編號：412

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本	試	題
共	/	頁
第	/	頁

1. X and Y are two discrete random variables with the joint PMF (probability mass function) $P_{X,Y}(x,y)$

$$P_{X,Y}(x,y) = \begin{cases} \frac{xy}{3025} & x=1,2,\dots,10, y=1,2,\dots,10 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Verify if X and Y are independent. (7%) (b) $Z = \max\{X, Y\}$. Find the PMF $P_Z(z)$ of Z . (8%)

2. U is a continuous random variable uniformly distributed on $(-\frac{\pi}{2}, \frac{\pi}{2})$. $X = \tan(U)$. Find the PDF (probability density function) $f_X(x)$ of X . (15%)

3. Roll a fair six-sided die 4 times. The outcome is independent from roll to roll. Let X_k = the outcome of the k -th roll, $k=1,2,3,4$, and thus the PMF $P_X(n)$ of X_k is

$$P_X(n) = \begin{cases} 1/6 & n=1,2,\dots,6 \\ 0 & \text{otherwise} \end{cases}$$

- (a) $Y = \min\{X_1, X_2, X_3, X_4\}$. Find the PMF $P_Y(n)$ of Y . (8%) (b) $Z = \max\{X_1, X_2, X_3, X_4\}$. Find the probability $P[Z > 4]$. (8%)

4. X and Y are two random variables with the joint PDF

$$f_{X,Y}(x,y) = \begin{cases} 4xy & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

- Let event $A = \{(X+Y) < 1\}$, and $Z = (X+Y)$. Find the conditional PDF $f_{Z|A}(z)$. (16%)

5. X_1 and X_2 are two joint Gaussian random variables. $E[X_1] = 2$, $E[X_2] = 1$, $\text{Var}[X_1] = 5$, $\text{Var}[X_2] = 3$, $E[X_1X_2] = -1$. Let $Y = (2X_1 + X_2)$. Find the PDF $F_Y(y)$ of Y . (10%)

6. Shuffle the 52 poker cards and 4 cards are dealt to you. Given that your first card is an Ace, find the probability that there are two or more Aces in the four cards dealt to you. (10%)

7. X_1, X_2, \dots, X_{100} are 100 iid (independent and identically distributed) exponential random variables each with the following PDF $f_X(x)$.

$$f_X(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

- Find the probability $P[(X_1 + X_2 + \dots + X_{100}) > 65]$. (18%)