

科目：微積分 適用：資工系二

編號：312

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題  
共 2 頁  
第 / 頁

The following problems may be answered in Chinese or English. You need to give all details in order to receive any credit (point).

1. (10%) Show that  $\lim_{x \rightarrow c} f(x) = L$  iff  $\lim_{x \rightarrow c} [f(x) - L] = 0$

2. (15%) Show that the function  $g(x, y) = \frac{x^2 y}{x^4 + y^2}$  has limiting value 0 as

$(x, y) \rightarrow (0, 0)$  along any line through the origin, but  $\lim_{(x, y) \rightarrow (0, 0)} g(x, y)$  still does not exist.

3. (10%) **Definition**: If  $z$  is irrational, then by  $e^z$  we mean the unique number

which has logarithm  $z$ , i.e.  $\ln e^z = z$ . Prove  $\frac{d}{dx}(e^x) = e^x$  for all real  $x$ .

4. (15%) A rod of length  $L$  is placed on the  $x$ -axis from  $x=0$  to  $x=L$ . Find the mass of the rod and the center of mass if the density of the rod varies directly as the distance from the  $x=0$  endpoint of the rod.

5. (10%) Find the volume of the solid generated by revolving the region between  $y = \sqrt{x}$ ,  $0 \leq x \leq 1$  and  $y = x^2$ ,  $0 \leq x \leq 1$ , around the line  $x = -2$ .

6. (15%)

- (a) (7%) Show that  $\sum_{k=0}^{\infty} \frac{k^k}{k!}$  diverges (Hint: by ratio test)

- (b) (8%) Let  $r$  be a positive number. For what values of  $r$  (if any) does

$\sum_{k=0}^{\infty} \frac{r^k}{k^r}$  converge? (Hint: by root test)

(Hint: a.  $\forall$  real  $x$ ,  $(1+x/n)^n \rightarrow e^x$  as  $n \rightarrow \infty$ . b.  $n^{1/n} \rightarrow 1$  as  $n \rightarrow \infty$ .)

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第 2 頁7. (25%) [ Hint : if  $f'(t)=kf(t)$ , then  $f(t)=f(0)e^{kt}$  ]

- (a) (2%) Find the gradient  $\nabla f(x,y)$ , where  $f(x,y) = x^2 + y^2$ .
- (b) (4%) At the point  $(1, 2, 5)$ , in what direction does  $f$  increase most rapidly? What is the magnitude of its speed?
- (c) (4%) Find the directional derivative of the function  $f$  at the point  $(1, 2)$  in the direction of the vector  $2\mathbf{i}-3\mathbf{j}$ .
- (d) (7%) Determine the path of steepest descent along the surface  $z = x^2 + y^2$  from the point  $(1, 2, 5)$ .
- (e) (3%) Determine the level curve of  $f$  that passes through the point  $(1, 2)$ .
- (f) (5%) Show that the gradient vector  $\nabla f(1, 2)$  is perpendicular to the level curve of  $f$  that passes through the point  $(1, 2)$ .

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