

科目：微積分 適用：資工系二

編號：312

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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The following problems may be answered in Chinese or English. You need to give all details in order to receive any credit (point).

1. (10%) Show that $\lim_{x \rightarrow c} f(x) = L$ iff $\lim_{x \rightarrow c} [f(x) - L] = 0$

2. (15%) Show that the function $g(x, y) = \frac{x^2 y}{x^4 + y^2}$ has limiting value 0 as

$(x, y) \rightarrow (0, 0)$ along any line through the origin, but $\lim_{(x, y) \rightarrow (0, 0)} g(x, y)$ still does not exist.

3. (10%) **Definition** : If z is irrational, then by e^z we mean the unique number

which has logarithm z , i.e. $\ln e^z = z$. Prove $\frac{d}{dx}(e^x) = e^x$ for all real x .

4. (15%) A rod of length L is placed on the x -axis from $x=0$ to $x=L$. Find the mass of the rod and the center of mass if the density of the rod varies directly as the distance from the $x=0$ endpoint of the rod.

5. (10%) Find the volume of the solid generated by revolving the region between $y = \sqrt{x}$, $0 \leq x \leq 1$ and $y = x^2$, $0 \leq x \leq 1$, around the line $x = -2$.

6. (15%)

(a) (7%) Show that $\sum_{k=0}^{\infty} \frac{k^k}{k!}$ diverges (Hint: by ratio test)

(b) (8%) Let r be a positive number. For what values of r (if any) does

$\sum_{k=0}^{\infty} \frac{r^k}{k^r}$ converge? (Hint: by root test)

(Hint: a. \forall real x , $(1+x/n)^n \rightarrow e^x$ as $n \rightarrow \infty$. b. $n^{1/n} \rightarrow 1$ as $n \rightarrow \infty$.)

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第 2 頁7. (25%) [Hint : if $f'(t)=kf(t)$, then $f(t)=f(0)e^{kt}$]

- (a) (2%) Find the gradient $\nabla f(x,y)$, where $f(x,y) = x^2 + y^2$.
- (b) (4%) At the point $(1, 2, 5)$, in what direction does f increase most rapidly? What is the magnitude of its speed?
- (c) (4%) Find the directional derivative of the function f at the point $(1, 2)$ in the direction of the vector $2\mathbf{i}-3\mathbf{j}$.
- (d) (7%) Determine the path of steepest descent along the surface $z = x^2 + y^2$ from the point $(1, 2, 5)$.
- (e) (3%) Determine the level curve of f that passes through the point $(1, 2)$.
- (f) (5%) Show that the gradient vector $\nabla f(1, 2)$ is perpendicular to the level curve of f that passes through the point $(1, 2)$.

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