Multiple Choice Questions (Total 100%, 5 points each)

(請務必將答案寫於答案卷首頁，違者不予計分。)

1. Find the area of the region R bounded by the curves $y=\sqrt{x}$, $y=\sqrt{2x}$, $y=\frac{x^2}{3}$, and $y=\frac{x^2}{4}$.

A) 1

B) $\frac{1}{3}$

C) $\frac{1}{2}$

D) $\frac{1}{4}$

2. Consider a wire of density $\rho(x,y,z)$ given by the space curve

$C: r(t)=x(t)i+y(t)j+z(t)k, \quad a\le t\le b,$

The moments of inertia about the $x-$, $y-$, and $z-$axes are given by

$I_x=\int_C (y^2+z^2)\,\rho(x,y,z)\,ds$

$I_y=\int_C (x^2+z^2)\,\rho(x,y,z)\,ds$

$I_z=\int_C (x^2+y^2)\,\rho(x,y,z)\,ds$

What are the moments of inertia for a wire of uniform density $\rho=1$ in the shape of the helix

$r(t)=3\cos ti+3\sin tj+2tk, \quad 0\le t\le 2\pi$

A) $I_x=\frac{1}{3}\sqrt{13}\pi(32\pi^2+27)$, $I_y=\frac{1}{3}\sqrt{13}\pi(32\pi^2+27)$, $I_z=18\pi\sqrt{13}$

B) $I_x=18\pi\sqrt{13}$, $I_y=\frac{1}{3}\sqrt{13}\pi(32\pi^2+27)$, $I_z=18\pi\sqrt{13}$

C) $I_x=\frac{1}{3}\pi(32\pi^2+27)$, $I_y=\frac{1}{3}\sqrt{13}\pi(32\pi^2+27)$, $I_z=\frac{1}{3}\sqrt{13}\pi(32\pi^2+27)$

D) $I_x=\frac{1}{3}\sqrt{13}\pi(32\pi^2+27)$, $I_y=18\pi\sqrt{13}$, $I_z=18\pi\sqrt{13}$

3. Electrical power P is given by

$$P = \frac{E^2}{R}$$

where E is voltage and R is resistance. What is the maximum percent error in calculating power if 200 volts is applied to a 4000-ohm resistor and the possible percent errors in measuring E and R are 2% and 3%, respectively.

A) 7%

B) 5%

C) 6%

D) 8%

4. The inductance L (in microhenrys) of a straight nonmagnetic wire in free space is

$$L = 0.0002l\left(\ln\frac{2h}{r} - 0.75\right)$$

where h is the length of the wire in millimeters and r is the radius of a circular cross section. Approximate L when $r = 2 \pm \frac{1}{16}$ millimeters and $h = 100 \pm \frac{1}{100}$ millimeters.

A) $L = 80.96 \times 10^{-4} \pm 6.6 \times 10^{-6}\ micro - henrys$

B) $L = 80.96 \times 10^{-4} \pm 6.4 \times 10^{-6}\ micro - henrys$

C) $L = 8.096 \times 10^{-4} \pm 6.4 \times 10^{-6}\ micro - henrys$

D) $L = 8.096 \times 10^{-4} \pm 6.6 \times 10^{-6}\ micro - henrys$

5. A company manufactures two types of wood-burning stoves: a freestanding model and a fireplace-insert model. The cost function for producing x freestanding and y fireplace-insert stoves is

$$C = 32\sqrt{xy} + 175x + 205y + 1050$$

Find the marginal costs $\frac{\partial C}{\partial x}$ and $\frac{\partial C}{\partial y}$ when $x = 80$ and $y = 20$

A) 237, 183

B) 173, 237

C) 183, 237

D) 183, 217

6. Find the following limit

$$\lim_{(x,y)\to(0,1)} \tan^{-1}\left[\frac{x^2+1}{x^2+(y-1)^2}\right]$$

A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) π

D) $\frac{\pi}{3}$

7. Find the curvature K of the curve, where s is the arc length parameter

$$r(s) = \left(1+\frac{\sqrt{2}}{2}s\right)i + \left(1-\frac{\sqrt{2}}{2}s\right)j$$

A) 1

B) 0

C) π

D) $\frac{1}{3}$

8. Consider the cornu spiral given by

$$x(t)=\int_0^t \cos\left(\frac{\pi u^2}{2}\right)du \quad \text{and} \quad y(t)=\int_0^t \sin\left(\frac{\pi u^2}{2}\right)du$$

Find the length of the cornu spiral from $t=0$ to $t=a$

A) a

B) $a+\pi$

C) 2π

D) $\frac{a+\pi}{2}$

9. Find the sum of the infinite series

$$\sum_{n=1}^{\infty}\frac{1}{n!(n+2)}$$

A) $\frac{1}{8}$

B) $\frac{1}{4}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

10. Find the sum of the infinite series

$$\sum_{n=0}^{\infty}\frac{(n+1)}{n!}$$

A) e

B) $2e$

C) $e+1$

D) $2e+2$

11. Consider the following sequence of numbers defined recursively.

$a_1=3,\ a_2=\sqrt{3},\ a_3=\sqrt{3+\sqrt{3}},\ \ldots,\ a_{n+1}=\sqrt{3+a_n}$

Please find the limit of the sequence.

A) $\frac{1+\sqrt{13}}{4}$

B) $\frac{1+\sqrt{15}}{2}$

C) $\frac{1+\sqrt{15}}{4}$

D) $\frac{1+\sqrt{13}}{2}$

12. Find the value of the positive constant c such that

$\lim_{x\to\infty}\left(\frac{x+c}{x-c}\right)^x=9$

A) $\ln(2)$

B) $\ln(3)$

C) $\ln(5)$

D) $\ln(7)$

13. Find the arc length of the graph of the function $y=\ln(1-x^2)$ on the interval $0\le x\le\frac{1}{2}$.

A) $\ln(2)-\frac{1}{2}$

B) $\ln(3)-\frac{1}{2}$

C) $\ln(2)-\frac{1}{4}$

D) $\ln(3)-\frac{1}{4}$

14. Find the arc length from $(0,3)$ clockwise to $(2,\sqrt{5})$ along the circle $x^2+y^2=9$.

A) $3\arcsin\frac{2}{3}$

B) $2\arcsin\frac{2}{3}$

C) $3\arcsin\frac{1}{3}$

D) $2\arcsin\frac{1}{3}$

15. The logistic equation $y=\frac{L}{1+be^{-kt}}$ can also be written as

A) $y=\frac{L}{4}\left[1+\tanh\left(\frac{1}{4}k\left(t-\frac{\ln b}{k}\right)\right)\right]$

B) $y=\frac{L}{4}\left[1+\tanh\left(\frac{1}{2}k\left(t-\frac{\ln b}{k}\right)\right)\right]$

C) $y=\frac{L}{2}\left[1+\tanh\left(\frac{1}{4}k\left(t-\frac{\ln b}{k}\right)\right)\right]$

D) $y=\frac{L}{2}\left[1+\tanh\left(\frac{1}{2}k\left(t-\frac{\ln b}{k}\right)\right)\right]$

16. Find the area under the curve

$y=\dfrac{1}{\sin^2 x+4\cos^2 x}$, between $x=0$ and $x=\dfrac{\pi}{4}$

A) $\dfrac{1}{2}\arctan\left(\dfrac{1}{2}\right)$

B) $\dfrac{1}{2}\arctan\left(\dfrac{1}{4}\right)$

C) $\dfrac{1}{4}\arctan\left(\dfrac{1}{2}\right)$

D) $\dfrac{1}{4}\arctan\left(\dfrac{1}{4}\right)$

17. Using an appropriate Riemann sum to evaluate the limit

$\lim\limits_{n\to\infty}\dfrac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}}{n^{3/2}}$

A) $\dfrac{1}{2}$

B) $\dfrac{1}{3}$

C) $\dfrac{2}{3}$

D) $\dfrac{1}{4}$

18. Find the following integral

$\int\dfrac{(\ln x)^2}{x}dx$, where C is a constant

A) $\dfrac{(\ln x)^3}{3}+C$

B) 0

C) $(\ln x)^3+C$

D) $\dfrac{(\ln x)^3}{2}+C$

19. Find the following integral

$$\int \arctan x\,dx$$

A) $x\arctan x+\frac{1}{2}\ln(1-x^2)+C$

B) $x\arctan x-\frac{1}{2}\ln(1+x^2)+C$

C) $x\arctan x+\frac{1}{2}\ln(1+x^2)+C$

D) $x\arctan x-\frac{1}{2}\ln(1-x^2)+C$

20. Find the following integral

$$\int \frac{u^2}{(a+bu)^2}du$$

A) $\frac{1}{b^3}\left(bu-\frac{a^2}{a+bu}-2a\ln|a+bu|\right)+C$

B) $\frac{1}{b^3}\left(bu+\frac{a^2}{a+bu}-a\ln|a+bu|\right)+C$

C) $\frac{1}{b^3}\left(bu-\frac{a^2}{a+bu}-a\ln|a+bu|\right)+C$

D) $\frac{1}{b^2}\left(bu-\frac{a^2}{a+bu}-2a\ln|a+bu|\right)+C$