國立暨南國際大學九十五學年度碩士班研究生入學考試試題 第1節

科目：工程數學　適用：通訊所

編號：451

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

本試題
共 1 頁
第 1 頁

1. (20%) Suppose a *geometric random variable* X has a probability mass function given by

$$p_X(k) = (1-p)^{k-1}p, \quad k = 1, 2, 3, \ldots,$$

show that

$$\sum_{k=1}^{\infty} p_X(k) = 1.$$

2. (20%) Let X be a random variable and let

$$Y = aX + b,$$

where a and b are given scalars. Show that

$$E[Y] = aE[X] + b, \quad var(Y) = a^2 \cdot var(X).$$

3. (20%) The *trace* of a square matrix A, denoted $tr(A)$, is the sum of the elements on the main diagonal of A. Show that, if A and B are $n \times n$ matrices:

   (a) $tr(AB) = tr(BA)$. (10%)

   (b) $tr(AA^T)$ is the sum of the squares of all entries of A. (10%)

4. (20%) Find the characteristic polynomial and eigenvalues for matrix A if

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

5. (20%) Find an orthonormal basis for the subspace of $\mathcal{R}^3$ consisting of all vectors (a, b, c) such that

$$a + b + c = 0.$$