

科目：微積分 適用：財金所

編號：354

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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1. Analyze and sketch a graph of the function. Label any intercepts, relative extrema, points of inflection, and asymptotes. (24%)

(a)  $f(x) = 3x^4 - 4x^3$ .

(b)  $f(x) = (x+3) + \frac{9}{x+3}$ .

(c)  $f(x) = \frac{(x+1)^3}{(x-1)^2}$ .

2. True or False? Determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false. (15%)

(a) If  $\lim_{x \rightarrow c} f(x) = L$  and  $f(c) = L$ , then  $f$  is continuous at  $c$ .(b) If  $p(x)$  is a polynomial, then the graph of the function given by

$$f(x) = \frac{p(x)}{x-1}$$
 has a vertical asymptote at  $x = 1$ .

(c) Consider the function  $f(x) = \sqrt{x}$ . Then  $\lim_{x \rightarrow 0.25} \sqrt{x} = 0.5$  and

$$\lim_{x \rightarrow 0} \sqrt{x} = 0.$$

3. Find the limit (if it exists). If it does not exist, explain why. (15%)

(a)  $\lim_{\Delta x \rightarrow 0^+} \frac{x + \Delta x}{\Delta x} - \frac{1}{x}$ .

(b)  $\lim_{x \rightarrow 3} (2 - [-x])$ , where  $[x]$  = greatest integer  $n$  such that  $n \leq x$ .

(c)  $\lim_{x \rightarrow 0} \frac{\int_0^{x^3} \sin t^2 dt}{x^9}$ .

4. Find the volume of the solid generated by revolving the region

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bounded by the graphs of the equations  $y = x^2$ ,  $y = 4x - x^2$  about the given lines (a) the  $x$ -axis, (b) the line  $y = 6$ . (10%)

5. Calculate: (20%)

(a)  $\int \frac{1 + \sin x}{1 - \cos x} dx$ .

(b)  $\int \ln \sqrt{x^2 - 1} dx$ .

(c)  $\int \frac{x^2 + 2x}{x^3 - x^2 + x - 1} dx$ .

(d)  $\int \frac{x^3}{\sqrt{4 + x^2}} dx$ .

6. Let  $R$  be the region bounded by the lines  $x - 2y = 0$ ,  $x - 2y = -4$ ,  $x + y = 4$ , and  $x + y = 1$ . (10%)

(a) Find a transformation  $T$  from a region  $S$  to  $R$  such that  $S$  is a rectangular region (with sides parallel to the  $u$ - or  $v$ -axis).

(b) Evaluate the double integral  $\iint_R 3xy \, dx \, dy$  by the indicated change of variables in (a).

7. Find the derivative of  $y = \frac{(\sin x + 1)^{1/3} (\tan x + 2)^{1/5}}{(\sec x + 3)^{1/7}}$ . (6%)