

科目：離散數學 適用：資工所

編號：413

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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The following problems may be answered in Chinese or English. You need to give all details in order to receive any credit (point).

**Problem 1 (15%)** Let  $G_0 = 3, G_1 = 4, G_{n+1} = G_n + G_{n-1}$  for  $n \geq 1$ . Prove that  $\sum_{k=0}^n G_k = G_{n+2} - 4$  for  $n \geq 5$ .

**Problem 2 (10%)** Prove that there exist  $k$  consecutive composite integers for every positive integer  $k$ .

**Problem 3 (15%)** Prove that a graph with  $n$  nodes and more than  $\binom{n-1}{2}$  edges is always connected. (Note: The graph here is simple and undirected.)

**Problem 4 (20%)** A graph  $G$  is called *planar* iff it can be drawn in the plane with its edges intersecting only at vertices of  $G$ . Let  $G$  be the graph obtained by omitting an edge of the complete graph  $K_5$  on five nodes. Prove or disprove that  $G$  is planar.

**Problem 5** Let  $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ . Let  $P(f, x_0)$  be the statement: for every  $\epsilon > 0$ , there exists  $\delta > 0$  such that  $|f(x) - f(x_0)| < \epsilon$  for all  $|x - x_0| < \delta$ . Here the universe comprises the positive real numbers.

- a. (10%) Express  $P(f, x_0)$  by the first-order logic.
- b. (10%) Describe its negating statement by the first-order logic.

**Problem 6 (20%)** Let  $f, g$  and  $h$  be functions from reals to reals. Prove that  $(f \circ g) \circ h = f \circ (g \circ h)$  where  $\circ$  is the composition of two functions.