國立暨南國際大學九十五學年度碩士班研究生入學考試試題

第 1 節

科目：工程數學甲　適用：電機所電子組

編號：421

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

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1. (30 pts.) Determine $y(t)$ by solving the following differential equations:

(a) (10 pts.) $y^{(4)}+2y''+y=0$, $y(0)=y'(0)=y''(0)=y'''(0)=0$.

(b) (10 pts.) $ty\dfrac{dy}{dt}+y^2=32t$, $y(-1)=0$.

(c) (10 pts.) $\begin{cases} x''+2x=y-x \\ y''+2y=x-y, \end{cases}$ $x(0)=2$, $x'(0)=y(0)=y'(0)=0$.

2. (20 pts.) Consider the two signals $y(t)=x(t)*h(t)$ and $z(t)=x(3t)*h(3t)$ (note that the symbol "$*$" is represented as the convolution integral), and assume that $X(j\omega)$ and $H(j\omega)$ are the Fourier transforms of $x(t)$ and $h(t)$.

(a) (10 pts.) Use Fourier transform properties to show that $z(t)$ can be represented in the form $z(t)=Ay(Bt)$, and determine the values of A and B.

(b) (10 pts.) If $x(t)=\dfrac{\sin t}{\pi t}$ and $h(t)$ is a unit impulse function, determine and sketch the Fourier transform of $z(t)$.

3. (20 pts.) Consider the following initial-value problem (IVP):

$$y'+ky=te^{-t},\quad y(0)=1,\quad k\in\Re.$$

(a) (10 pts.) Find the Laplace transform of the solution for this IVP.

(b) (10 pts.) Determine the solution of this IVP by finding the inverse Laplace transform of part **(a)**.

4. (30 pts.) True or false? Justify you answers. (Give a brief proof or explanation if it is true. Otherwise, give a right correction or counter-example to show that it is false).

(a) (6 pts.) The function $f_1(t)$ is equal to the function $f_2(t)$ if and only if their Laplace transforms are identical.

(b) (6 pts.) A constant multiple of a solution of a linear differential equation is also a solution.

(c) (6 pts.) If the Laplace transform of $f(t)$ is $F(s)$, then the Laplace transform of the function $e^{at}\int_0^t f(\tau)\,d\tau$ can be written as $\dfrac{F(s-a)}{s}$.

(d) (6 pts.) The Fourier transforms of real and even signals are also real and even.

(e) (6 pts.) The boundary-value problem $y''-a^2y=0$, $y(0)=y(\frac{\pi}{2})=0$, where $a\in\Re$, possesses only the trivial solution.