

科目：工程數學 適用：土木所耐震 土木所應力 地震所

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題

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編號：461 481 491

- 1 Please solve the following initial value problem. (20%)

$$x^2 y'' + xy' + 4y = 0 \quad \text{with} \quad y(1) = 2, \quad -y'(1) = -1 \quad \text{and} \quad x > 0.$$

- 2 An unit step function is defined as
- $u(t-a) = \begin{cases} 0 & \text{if } t < a \\ 1 & \text{if } t > a \end{cases}$

For the following initial value problem,  $y'' + 3y' + 2y = r(t)$ ,

$$\text{with } r(t) = \begin{cases} 2 & \text{if } 1 < t < 2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad y(0) = y'(0) = 0 \quad \text{please}$$

(a) express  $r(t)$  with unit step function (10%), and(b) find the solution  $y(t)$  by means of Laplace Transformation. (15%)

$$\text{Remark: } \mathcal{L}[u(t-a)] = e^{-as}/s, \quad \mathcal{L}[e^{at}f(t)] = F(s-a)$$

$$\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s), \quad \mathcal{L}\left[\frac{t^{n-1}}{(n-1)!}\right] = 1/s^n \quad n=1,2,\dots$$

- 3 Please find the real general solution
- $\mathbf{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$
- of the following system

$$\begin{cases} y_1' = y_1 + y_2 \\ y_2' = 3y_1 - y_2 \end{cases} \quad (20\%).$$

- 4 Let
- $\mathcal{F}(f)$
- denote the fourier transform of
- $f$
- , that
- $\mathcal{F}(f) = \hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-iwx} dx$
- .

Please find the fourier transform of  $f(x) = k$  if  $0 < x < a$  and  $f(x) = 0$  otherwise. (20%)

- 5 For a real and symmetric matrix,
- $\mathbf{A}$
- , please show that its eigenvectors associated with distinct eigenvalues are orthogonal. (15%)