

科目：機率 適用：資工所

編號：412

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

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第 1 頁

1. X_1, X_2, \dots, X_{150} are 150 iid (independent and identically distributed) random variables each with the following PDF (probability density function),

$$f_X(x) = \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability $P[|X_1 + X_2 + \dots + X_{150}| \geq 12]$. (12%)

2. Shuffle the 52 poker cards and 2 cards are dealt to you. X is the number of the Kings in the 2 cards. Y is the number of the red cards in the 2 cards. Find the probability $P[X = Y]$. (12%)

3. Roll a fair six-sided die 12 times. The outcome is independent from roll to roll, and is equally likely to be 1 through 6. Let X_k = the outcome of the k -th roll, $k = 1, 2, \dots, 12$. $Y_1 = \max\{X_1, X_2, X_3\}$, $Y_2 = \max\{X_4, X_5, X_6\}$, $Y_3 = \max\{X_7, X_8, X_9\}$, $Y_4 = \max\{X_{10}, X_{11}, X_{12}\}$, and $Z = \min\{Y_1, Y_2, Y_3, Y_4\}$. Find the probability $P[Z = 4]$. (18%)

4. N is the number of the customers entering the 7-11 convenience store at NCNU from 06:00 through 07:00 every day, and N is a Poisson random variable with $E[N] = 6$. Assume the arrival rate of the customers remains a constant during the 60 minutes. (a) Find the probability that two or more customers enter the 7-11 between 06:00 and 06:30. (5%) (b) Given that no customer shows up between 06:15 and 06:30, find the probability that no customer enters the 7-11 from 06:30 through 06:50, either. (7%) (c) For each of the N customers entering the 7-11 between 06:00 and 07:00, the probability that she or he spends less than 100 dollars is 0.75. Let K = the number of the customers who spends less than 100 dollars between 06:00 and 07:00. Find the joint PMF $P_{K,N}(k, n)$. (8%) (d) Find $E[K]$. (6%)

5. X and Y are two random variables with the joint PDF

$$f_{X,Y}(x, y) = \begin{cases} 0.5 & 0 < x < 1, -1 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Let event $A = \{(X^2 + Y^2) < 1\}$, $R = \sqrt{X^2 + Y^2}$, $W = \tan^{-1}(\frac{Y}{X})$. Find the conditional joint PDF $f_{R,W|A}(r, w)$ and verify if $f_{R|A}(r)f_{W|A}(w) = f_{R,W|A}(r, w)$. (18%)

6. X and Y are two joint Gaussian random variables. $E[X] = 0$, $E[Y] = 3$, $\text{Var}[X] = 4$, $\text{Var}[Y] = 16$, $E[XY] = E[X]E[Y] = 0$. Find (a) $E[X^2Y^2]$ (7%) (b) $E[X^3Y^4]$ (7%)