科目：數學(以離散數學、線性代數為主)

編號：343 適用：資工系

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

(以下各題均須寫出計算或證明過程方予計分)

1. Determine each of the following:
   (2%) (a) $\lfloor 2.3 - 1.6 \rfloor$
   (2%) (b) $\lfloor 2.3 \rfloor - \lfloor 1.6 \rfloor$
   (2%) (c) $\lceil 2.3 \rceil \lfloor -1.6 \rfloor$

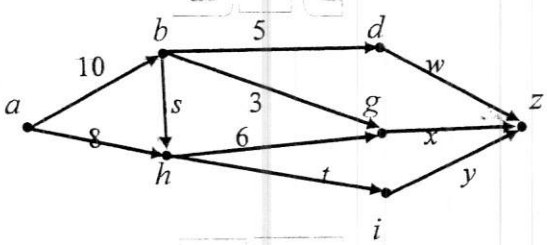
2. (5%) In how many ways can one arrange three 1's and three −1's so that all six partial sums (staring with the first summand) are nonnegative?

3. (10%) Prove that: for each $n \in \mathbf{Z}^+$, a sequence of $n^2 + 1$ distinct real numbers contains a decreasing or increasing subsequence of length $n + 1$.

4. (10%) Determine the coefficient of x^8 in $\frac{1}{(x-3)(x-2)^2}$.

5. (10%) For $n \in \mathbf{Z}^+$, determine the number of ways one can tile a $1 \times n$ chessboard using 1×1 white (square) tiles and 1×2 blue (rectangular) tiles.

6. (5%) (a) For the following network, let the capacity of each edge be 10. If each e in the figure is labeled by a function f, as shown, determine the values of s, t, w, x, and y so that f is a flow in the network.
   (1%) (b) What is the value of this flow?
   (3%) (c) Find three cuts $(P, \bar{P})$ in this network that have capacity 30.



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7. (10%) For the Euclidean vector space R^3:
   (a) (3%) Find a subset $\boldsymbol{S}$ of R^3 with four vectors such that $\boldsymbol{S}$ can span R^3.
   (b) (2%) Find a linear independent subset $\boldsymbol{S}$ of R^3 such that $\boldsymbol{S}$ can span R^3.
   (c) (2%) Find an orthonormal basis.
   (d) (3%) Explain that why any linear independent subset of R^3 with two vectors cannot span R^3. Take an example of dimension 2.

8. (10%) Explain that why the Gaussian elimination procedure can be used to exactly solve any system of linear equations. (That is, why the equivalent systems of the linear equations have exactly the same solutions.)

9. (7%) For the vector space R^2 with the inner product $<\mathbf{u}, \mathbf{v}> \triangleq \frac{1}{9} u_1 v_1 + \frac{1}{4} u_2 v_2$, where $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$, find and sketch the unit circle.

10. (8%) Let $W = \{t(1, 2, 3), t \in R\}$, find its orthogonal complement in R^3.

11. (15%) For the Euclidean vector space R^2, let $B = \{(1, 0), (0, 1)\}$ and $B' = \{(\sqrt{2}/2, \sqrt{2}/2), (\sqrt{2}/2, -\sqrt{2}/2)\}$ respectively be the standard basis and the new basis of the two coordinate systems, $x - y$ and $x' - y'$, as shown in Fig. 1.
   (a) (5%) Find the transition matrix A from B to B' such that $A[\mathbf{v}]_B = [\mathbf{v}]_{B'}$, where $[\mathbf{v}]_B$ and $[\mathbf{v}]_{B'}$ respectively denote coordinate vectors of $\mathbf{v}$ relative to B and B'.
   (b) (5%) Let $\mathbf{v} = (2, -2)$, find $[\mathbf{v}]_{B'}$.
   (c) (5%) Find the null space of A.

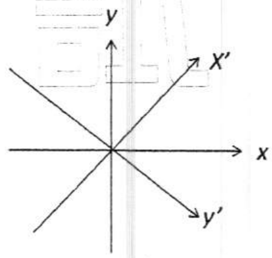

Fig. 1 The standard coordinate system $x - y$ and the new coordinate system $x' - y'$