

科目：數學 適用：資工系

編號：412

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題

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第 1 頁

(以下各題均須寫出計算過程方予計分)

1. (2%) Calculate  $C(7, 3)$ .

2. (10%) Prove: there are infinitely many primes.

3. (8%) A rumor is spread as follows. The originator calls two people. Each of these people phones three friends, each of whom in turn calls five associates. If no one receives more than one call, and no one calls the originator,

(a) (4%) how many people now know the rumor?

(b) (4%) How many phone calls were made?

4. (10%) Mrs. Blasi has five sons (Michael, Rick, David, Kenneth and Donald) who enjoy reading books about sports. With Christmas approaching, she visits a bookstore where she finds 12 different books on sports.

(a) (2%) In how many ways can she select nine of these books?

(b) (4%) Having made her purchase, in how many ways can she distribute the books among her sons so that each of them gets at least one book?

(c) (4%) Two of the nine books Mrs. Blasi purchased deal with basketball, Donald's favorite sport. In how many ways can she distribute the books among her sons so that Donald gets at least the two books on basketball?

5. (a) (4%) For what sequence of numbers is  $g(x) = (1 - 2x)^{-5/2}$  the exponential generating function?(b) (6%) Find  $a$  and  $b$  so that  $(1 - ax)^b$  is the exponential generating function for the sequence  $1, 7, 7 \cdot 11, 7 \cdot 11 \cdot 15, \dots$ 6. (10%) Use a recurrence relation to derive the formula for  $\sum_{i=0}^n i^2$ .

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7. (10%) Find the determinant of the matrix  $A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$ .

8. (10%) Find the rank and nullity of the matrix  $A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$ .

9. (30%) Let  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$  and  $\mathbf{b} = (6, 0, 9)$ .

- (a) (5%) Find an orthonormal basis  $\{\mathbf{u}_1, \mathbf{u}_2\}$  for the column space of  $A$ .
- (b) (5%) Find the orthogonal projection of  $\mathbf{b}$  on the column space of  $A$ .
- (c) (5%) Find the orthogonal complement of the column space of  $A$ .
- (d) (5%) Find a vector  $\mathbf{u}_3$  in the Euclidean space  $\mathbf{R}^3$ , such that  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is an orthonormal basis for  $\mathbf{R}^3$ .
- (e) (5%) Find the new coordinate vector of  $\mathbf{b}$  relative to the orthonormal basis obtained in (d).
- (f) (5%) Verify that the norm (length) of  $\mathbf{b}$  can also be calculated from its new coordinate vector found in (e).