

科目：工程數學 適用：土木系(大地、水利及防災組)

編號：434

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題  
共壹頁  
第壹頁

1. (20%) Solve the ordinary differential equation:

$$y' = -\frac{y \cos t}{1+2y^2}, y(0) = 1$$

2. (10%)  $J_n(x)$  and  $Y_n(x)$  are the first kind and second kind of Bessel functions of order  $n$ , where  $n$  is an integer. For a Bessel equation  $x^2 y'' + xy' + (x^2 - n^2)y = 0$ , explain that the general solution is  $y(x) = c_1 J_n(x) + c_2 Y_n(x)$  rather than  $y(x) = c_1 J_n(x) + c_2 J_{-n}(x)$ .

3. (20%) Find the eigenvalues and eigenvectors of the following matrix:

$$\begin{bmatrix} 7 & 0 & -10 \\ 5 & 2 & -10 \\ 5 & 0 & -8 \end{bmatrix}$$

4. (20%) Solve the linear system of equations

$$2x_1 + x_2 - x_3 + 2x_4 = 7$$

$$-x_1 + 2x_2 - 3x_3 + x_4 = 2$$

$$x_1 - 3x_2 + x_3 - 2x_4 = -1$$

$$3x_1 - 2x_2 - 2x_3 + x_4 = 9$$

5. (10%) Find  $\mathcal{L}^{-1}\left\{\frac{9s+1}{s^2+4s-5}\right\}$ 6. A scalar function  $f(x,y,z) = (x+y+z)^2$ , find(a) (10%)  $\nabla \cdot \nabla f$ (b) (10%) the value of the line integral:  $\oint_C \nabla f \cdot d\vec{r}$ , where  $C: x^2 + y^2 = 1, z = 0$ ,

$$\vec{r}(x,y,z) = x\vec{i} + y\vec{j} + z\vec{k}$$