

科目：工程數學 適用：應光系

編號：421

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

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1. (15%) Find the directional derivative of  $f(x, y) = 4 - x^2 - \frac{1}{2}y^2$ , at  $(1, 2)$  in the direction of  $\mathbf{u} = \cos\left(\frac{\pi}{3}\right)\hat{x} + \sin\left(\frac{\pi}{3}\right)\hat{y}$

2. (15%) Solve  $\frac{dy}{dx} = y^2 - 4$

3. (15%) Solve  $\cos x (e^{2y} - y) \frac{dy}{dx} = e^y \sin 2x$ ,  $y(0) = 0$

4. (15%) Solve  $y'' + y = 4x + 10 \sin x$ ,  $y(\pi) = 0, y'(\pi) = 2$

5. (20%) Please use Laplace Transform and the following table to solve the differential equation

For  $y = y(t)$

$$y'' - 3y' + 2y = e^{-4t}, y(0) = 1, y'(0) = 5$$

$f(t)$	$L(f)$
1	$1/s$
$e^{at}$	$1/(s - a)$
$\cos(\omega t)$	$s/(s^2 + \omega^2)$
$\sin(\omega t)$	$\omega/(s^2 + \omega^2)$

6. (20%) Find the Fourier integral representation of the piecewise-continuous function

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & 0 < x < 2 \\ 0, & x > 2 \end{cases}$$