1) Answer True or False for each statement below:

1a) **(3%)** If f is a strictly decreasing function, which means $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$, then it cannot be an even function.

1b) **(3%)** If $\lim_{x \to a^-} f'(x)$ exists but $\lim_{x \to a^+} f'(x)$ does not exist, then we say that $f(x)$ is differentiable at a "on the left".

1c) **(3%)** Let f be a continuous function on the closed interval $[-1, 1]$ and differentiable on the open interval $(-1, 1)$. Suppose $f(-1) = 4$ and $f(1) = 3$. Then, there must exist $c \in (-1, 1)$ such that $f'(c) = 0$.

1d) **(3%)** Suppose f is differentiable on $\mathbb{R}$ and n is an integer. Let $F(x) = f(x^n)$. Then, by the power rule, $F'(x) = nf'(x^{n-1})$.

1e) **(3%)** Suppose f is differentiable on $\mathbb{R}$ and n is an integer. Let $F(x) = f(nx)$. Then, by the chain rule, $F'(x) = nf'(nx)$.

2) Answer True or False for each statement below:

2a) **(3%)** If $F(x)$ is differentiable on $\mathbb{R}$, then $\int_{-x}^{x} F'(v)dv = F(x) - F(-x)$ for any $x \in \mathbb{R}$.

2b) **(3%)** If $F(\cdot)$ is differentiable on $\mathbb{R}$, then by substitution,

$$\int_a^b 2xF'(x^2)dx = [F(b) - F(a)].$$

2c) **(3%)** If f and g are continuous on $[a, b]$, then

$$\int_a^b [\alpha f(x) + \beta g(x)]\,dx = \alpha \int_a^b f(x)dx + \beta \int_a^b g(x)dx\,.$$

2d) **(3%)** If f is continuous and even on $\mathbb{R}$, then we always have $\int_{-a}^{a} f(x)dv = 0$.

2e) **(3%)** If f is continuous, then for any a, b, and c,

$$\frac{d}{dx}\left(\int_a^x f(u)du\right) = \frac{d}{dx}\left(\int_b^x f(u)du\right).$$

3) (10%) Answer A, B or C: If $y = f(u)$ and $u = g(x)$, where f and g are twice differentiable such that $g'(x) = g(x)$, then $\frac{d^2y}{dx^2}$ equals ...

A. $\frac{d^2y}{dx^2}u + \frac{dy}{dx}u$,　　B. $\frac{d^2y}{dx^2}u + \frac{dy}{dx}u^2$,　　C. $\frac{d^2y}{dx^2}u^2 + \frac{dy}{dx}u$.

4) (5%) Answer A, B, C or D: If x is measured in meters, $r(x)$ is a function of x and r is also measured in meters, and ρ is measured in kg.m , then what is the unit of $\int_0^L \pi \rho r^3(x)\, dx$?

A. kg.m^{-1},　B. kg.m,　C. kg,　D. kg.m^2.

5) (10%) Answer A, B, C or D: Which of the following expressions is WRONG? If you think they are all correct, answer D.

A. $\int_a^b f'(g(x))\, g'(x) dx = f(g(b)) - f(g(a))$

B. $\int_a^b f(g'(x)) dx = f(g(b)) - f(g(a))$

C. $\int_{g(a)}^{g(b)} f'(x) dx = f(g(b)) - f(g(a))$

D. None of the above.

6) (15%) Show all derivation steps: Find $\int_0^\infty R(t)\, dt$ where $R(t) = \frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}}$.

7) (15%) Show all derivation steps: Evaluate the infinite integral $\int_1^\infty \frac{(\ln x)^2}{x^2} dx$ using L'Hôpital's rule.

8) (15%) Show all derivation steps: Find the following limit using Taylor series expansion:

$$\lim_{x \to 0} \frac{\sin x - x}{x}.$$

END OF QUESTIONS