國立暨南國際大學 106 學年度碩士班入學考試試題　　第 1 節

科目：工程數學三(機率)　　適用：電機系(通訊工程)

編號：363

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題
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1. (20%) Find the mean value and the variance of each of the random variables X_1 and X_2 defined as follows.

   (1) (10%) X_1 having a probability density $f_{X_1}(x) = \begin{cases} \frac{1}{b}e^{(x-a)/b} & x \geq a \\ 0 & x < a \end{cases}$

   with $b > 0$.

   (2) (10%) X_2 having a probability density $f_{X_2}(x) = \begin{cases} \frac{5}{4}(1 - x^4) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$.

2. (20%) Let $Z = (aX + 2Y)^2$ where X and Y are zero-mean random variables with variances $\sigma_X^2 = 1$ and $\sigma_Y^2 = 4$. The correlation coefficient of X and Y is $\rho_{XY} = -0.5$.

   (1) (10%) Find a value for the parameter a that minimizes the mean value of Z.

   (2) (10%) According to (1), find the minimum mean value of Z.

3. (20%) The random variables X and Y are uniformly distributed inside a triangle in the $x - y$ plane. which is $\{(x, y)|(x + y) \leq 5, x > 0 \text{ and } y > 0\}$.

   (1) (6%) Find the joint probability density function of X and Y.

   (2) (7%) Find the marginal probability density function of X.

   (2) (7%) Find the conditional probability density function of X given Y.

4. (20%) Assume that each of a series of random variables X_1, X_2, ..., X_N is uniformly distributed on the interval $[-0.5, 0.5]$.

   (1) (10%) Find the probability density function of $Y = X_1 + X_2$.

   (2) (10%) Find the probability density function of
   $Z = \lim_{N \to \infty} \frac{1}{\sqrt{N}}(X_1 + X_2 + ... + X_N)$.

5. (20%) A Gaussian random variable X has a probability density function

$$f_X(x) = \frac{1}{3\sqrt{2\pi}}e^{-\frac{(x-2)^2}{18}}.$$

   (1) (7%) Find the probability density function of $Y = 5X + 7$.

   (2) (7%) Find the probability density function of $Z = \left(\frac{X-2}{3}\right)^2$.

   (3) (6%) Given that $W = (X - 1)^3$, find the probability value $P(W \leq 1)$.