

## 科目：數學(以離散數學、線性代數為主)

編號：343

適用：資工系

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

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(以下各題均須寫出計算或證明過程方予計分)

1. Evaluate each of the following.

(5%) (a)  $P(8, 3)$ (5%) (b)  $C(10, 4)$ 2. (10%) Show that the equation  $y = \frac{\ln(\frac{x}{m} - as)}{r^2}$  is equal to  $me^{ry} = x - mas$ .3. (10%) In  $n \in \mathbf{Z}^+$ , and  $n$  is odd, prove that  $8 \mid (n^2 - 1)$ .4. (10%) Let  $m \in \mathbf{Z}^+$  with  $m$  odd. Prove that there exists a positive integer  $n$  such that  $m$  divides  $2^n - 1$ .

5. If a 20-digit ternary (0, 1, 2) sequence is randomly generated, what is the probability that:

(a) (5%) It has an even number of 1's?

(b) (5%) The total number of 0's and 1's is odd?

6. (8%) Evaluate the inverse of the following matrix:

$$\begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}.$$

7. (8%) Compute the following determinant:

$$\begin{vmatrix} 3 & 1 & 2 \\ 4 & 5 & 1 \\ -1 & 2 & -3 \end{vmatrix}.$$

8. (8%) Find the rank of the following matrix:

$$\begin{pmatrix} 2 & 0 & 1 \\ -5 & 1 & 2 \\ 4 & -2 & -7 \end{pmatrix}.$$

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9. (8%) Let  $A_1, \dots, A_k$  be  $n \times n$  nonsingular matrices for positive integers  $k$  and  $n$ .  
Prove that  $(A_1 \times A_2 \times \dots \times A_k)^{-1} = A_k^{-1} \times \dots \times A_2^{-1} \times A_1^{-1}$ .

10. (8%) Find the linear transformation that can map  $(1, 0, 1)^t$  to  $(2, 1, 0)^t$ ;  $(0, 2, 1)^t$  to  $(0, 0, 1)^t$ ; and  $(2, 0, 1)^t$  to  $(6, 2, 1)^t$ .

11. (10%) Evaluate the value of the following expression:

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^{20}$$