

科目：離散數學

適用：資工系三

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本 試 題

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1、是非題 (3% × 5)

- (a) All people who are concerned about the environment recycle their plastic containers. Margarita is not concerned about the environment. Therefore Margarita does not recycle her plastic containers.

Consider the open statement: $p(x, y): y - x = y + x^2$, where the universe for each of the variables x, y comprises all integers. Determine the truth value for (b) and (c).

(b) $\forall y p(0, y)$

(c) $\forall y \exists x p(x, y)$

- (d) If B is infinite and $A \subseteq B$, then A is infinite.

- (e) Let $A, B, C \subseteq U$, then $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $C \subseteq A$.

簡答題 (3% × 5)

2. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, determine the following:

(a) $|A \times B|$;

- (b) The number of relations from A to B that contain $(1, 3)$ and $(1, 5)$;

- (c) The number of relations on A that contain at least five elements;

- (d) How many functions are there from A to B ? How many of these are one-to-one? How many are onto?

- (e) How many functions $f: A \rightarrow B$ are such that $f^{-1}(\{4, 5\}) = \{1, 2\}$?

計算與證明題 (70%) (以下各題均須寫出計算或證明過程方予計分)

3. (10%) Find the values of sum after the given program segment is executed. (Here i, j, k , $increment$, and sum are integer variables.)

```
increment := 0
```

```
sum := 0
```

```
for i := 1 to 10 do
```

```
  for j := 1 to i do
```

```
    for k := 1 to j do
```

```
      begin
```

```
        increment := increment + 1
```

```
        sum := sum + increment
```

```
      end
```

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4. (15%) A penny is tossed 60 times yielding 45 heads and 15 tails. In how many ways could this have happened so that there were no consecutive tails?
5. (15%) How many permutations of the 26 different letters of the alphabet contain neither the pattern "MAN" nor the pattern "ANT"?
6. (15%) A sequence of numbers a_1, a_2, a_3, \dots is defined by
 $a_1 = 1 \quad a_2 = 2 \quad a_n = a_{n-1} + a_{n-2}, n \geq 3.$
(a) (5%) Determine the values of a_3, a_4, a_5, a_6 , and a_7 .
(b) (10%) Prove that for all $n \geq 1, a_n < (7/4)^n$.
7. (15%) Let $k \in \mathbb{Z}^+$. Prove that there exists a positive integer n such that $k|n$ and the only digits in n are 0's and 3's.

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