

科目：工程數學(線性代數+微分方程) 適用：電機系三

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

編號：732

本 試 題
共 1 頁
第 1 頁

❖ 請務必列出計算過程，否則不予計分。

1. (a) Find a basis for the matrix A . (5%)
(b) Find the condition, with which the rank of the matrix A is 2. (5%)

$$A = \begin{bmatrix} 1 & 1 & a \\ 1 & a & 1 \\ a & 1 & 1 \end{bmatrix}$$

2. (a) Find the eigenvalues and eigenvectors for the matrix B . (14%)

$$B = \begin{bmatrix} 1 & 0 & 4 \\ -2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

- (b) Find the algebraic multiplicity, geometric multiplicity, and defect of each eigenvalue. (6%)

3. (a) Find the eigenbasis and diagonalize the matrix C . (10%)

$$C = \begin{bmatrix} 6 & 4 \\ -10 & -8 \end{bmatrix}$$

- (b) Calculate C^3 , based on the method of matrix diagonalization. (10%)

4. Solve the following ODEs (50%; each 10%)

- (a) $y'' + 9y = 15e^x$

- (b) $y''' - y'' + 100y' - 100y = 0$

- (c) $y' + xy = xy^{-1}$

- (d) $y^{iv} - 5y'' + 4y = 10e^{-3x}$; $y'''(0) = y''(0) = y'(0) = y(0) = 0$

- (e) $\begin{cases} y_1' = -y_1 - y_2 \\ y_2' = y_1 - y_2 \end{cases}$; $y_1(0) = 0, y_2(0) = 1$