科目：離散數學　適用：資工系三

編號： 812

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

(以下各題均須寫出計算或證明過程方予計分)

1. In the following program segment, i, j, k and *counter* are integer variables. Determine the value that the variable *counter* will have after segment is executed. (10%)

```
counter := 10
for i := 1 to 15 do
  for j := i to 15 do
    for k := j to 15 do
      counter := counter + 1
```

2. Determine whether each of the following statements is true or false. If true, give the proof. If false, provide a counterexample. The universe comprises all integers.
   a) $\forall x\, \exists y\, \exists z\, (x = 7y + 5z)$ (5%)
   b) $\forall x\, \exists y\, \exists z\, (x = 4y + 6z)$ (5%)

3. If three integers are selected, at random and without replacement, from $\{1, 2, 3, \ldots, 99, 100\}$, what is the probability their sum is even? (10%)

4. A wheel of fortune has the integers from 1 to 25 places on it in a random manner. Show that regardless of how the numbers are positioned on the wheel, there are three adjacent numbers whose sum is at least 39. (10%)

5. Determine all $a, b \in \mathbf{Z}$ such that $\frac{a}{7} + \frac{b}{12} = \frac{1}{84}$. (10%)

6. Prove that if $f: A \to B$ and $B_1, B_2 \subseteq B$, then
   a) $f^{-1}(B_1 \cap B_2) = f^{-1}(B_1) \cap f^{-1}(B_2)$; (5%)
   b) $f^{-1}(B_1 \cup B_2) = f^{-1}(B_1) \cup f^{-1}(B_2)$. (5%)

7. A relation $\mathcal{R}$ on a set A is called *irreflexive* if for all $a \in A$, $(a, a) \notin \mathcal{R}$.
   a) Give an example of a relation $\mathcal{R}$ on $\mathbf{Z}$ where $\mathcal{R}$ is irreflexive and transitive but not symmetric. (4%)
   b) Let $\mathcal{R}$ be a nonempty relation on a set A. Prove that if $\mathcal{R}$ satisfies any two of the following properties — irreflexive, symmetric, and transitive — then it cannot satisfy the third. (10%)
   c) If $|A| = n \geq 1$, how many different relations on A are irreflexive? How many are neither reflexive nor irreflexive? (6%)

8. For the positive integers $1, 2, 3, \ldots, n+1, n$, there are 11,660 derangements where 1, 2, 3, 4, and 5 appear in the first five positions. What is the value of n? (10%)

9. Sergeant Bueti must distribute 40 bullets (20 for rifles and 20 for handguns) among four police officers so that each officer gets at least two, but no more than seven, bullets of each type. In how many ways can he do this? (10%)