科目：工程數學(線性代數+微分方程)　適用：電機系三

編號： 842

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

1. (10%) For each matrix below, find the eigenvalues and the corresponding eigenvectors each with a unit length.

   (a) (5%) $A = \begin{bmatrix} 1 & 2 & 2 \\ -1 & -2 & -1 \\ 1 & 1 & 0 \end{bmatrix}$.　(b) (5%) $A = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -2 & 0 \\ 3 & -5 & -2 \end{bmatrix}$.

2. (15%) Find the complete solution to $A\mathbf{x} = \mathbf{b}$.

   (a) (8%) $A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$.

   (b) (7%) $A = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 6 & 9 \\ -2 & -6 & 6 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 5 \\ 10 \end{bmatrix}$.

3. (15%) Find the projection of $\mathbf{b}$ onto the column space of A.

   (a) (8%) $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$.

   (b) (7%) $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 7 \\ 4 \\ 3 \end{bmatrix}$.

4. (10%) True or false.

   (a) (2%) An invertible matrix is always diagonalizable.

   (b) (2%) Two similar matrices share the same eigenvalues and eigenvectors.

   (c) (2%) The rank of a matrix A equals the number of independent rows in A.

   (d) (2%)A real symmetric matrix is always diagonalizable.

   (e) (2%) If A and B are similar invertible matrices, then A^{-1} and B^{-1} are similar.

5. (10%) Find the Laplace transforms of the following functions:

   (a) (5%) $\sin^2(2t) + te^{3t}$.

   (b) (5%) $e^t * \cos(3t)$, where "$*$" denotes the convolution operation.

6. (10%) Find the inverse Laplace transforms of the following functions:

   (a) (5%) $(\frac{1}{s^2} + \frac{1}{s^3})e^{-2s}$.

   (b) (5%) $\frac{\pi}{s^2(s^2+16)}$.

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7. (15%) Solve the following initial value problems:

(a) (5%) $y\dfrac{dy}{dx} + 4x = 0, y(0) = 3.$

(b) (5%) $(x^4 + y^2)dx - (xy)dy = 0, y(2) = 1.$

(c) (5%) $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0, y(0) = 4, y'(0) = -6.$

8. (15%) Find a general solution for each of the following ordinary differential equations (ODEs):

(a) (5%) $\dfrac{d^2y}{dx^2} - 16y = 19.2e^{4x} + 60e^{x}.$

(b) (5%) $2x^2\dfrac{d^2y}{dx^2} + 4x\dfrac{dy}{dx} + 5y = 0.$

(c) (5%) $(e^y - ye^x)dx + (xe^y - e^x) = 0$

試題