科目：微積分　適用：資工系二、土木系二、應化系二、電機系二

編號：412、421、433、441

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

[請寫出計算或推導過程，否則不予計分]

1. Prove the following statements using the ϵ, δ definition of limit.
   (a). $\lim_{x\to 3} x^2 = 9$ (5%)
   (b). $\lim_{x\to 0} \sin(x) = 0$ (5%)
2. Use the continuity to evaluate $\lim_{x\to\pi} \sin(x + \sin(x))$. (5%)
3. Evaluate the limit, if it exists:
   (a). $\lim_{x\to 2} \frac{x^2-3x+2}{x-2}$ (5%)
   (b). $\lim_{x\to 0} \frac{(1+x)^2-1}{x}$ (5%)
4. Find the derivatives of the following functions.
   (a). $f(x) = \frac{x}{2-\tan(x)}$ (5%)
   (b). $f(\theta) = \sin(\theta)$ where θ is measured in degrees. (5%)
5. Evaluate the following integrals.
   (a). $\int_{-1}^{1} \frac{e^{tan^{-1}(y)}}{1+y^2} dy$ (10 %)
   (b). $\int \sqrt{x} e^{\sqrt{x}} dx$ (10%)
   (c). $\int_{2}^{3} \frac{t^3+1}{t^2-t^3} dt$ (10%)
6. Find the sums of the following series:
   (a). $3 + \frac{4}{2!} + \frac{8}{3!} + \frac{16}{4!} + \cdots$ (5%)
   (b). $\sum_{n=0}^{\infty} \frac{3^n}{5^n n!}$ (5%)
7. Sketch the curve $r(\theta) = 2 - \sin(\theta)$ (5%) and find the area that it encloses in the polar coordinate system (10%).
8. Find the length of the curve in 3-D space

$$x(t) = 3\sin(t)$$
$$y(t) = 4t$$
$$z(t) = 3\cos(t)$$

where $-10 \le t \le 10$. (10%)