科目：線性代數 適用：電機系(系統組、通訊工程碩士班)

編號：471, 481

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

1. (20%) Answer the following questions:
   (a) (5%) What matrix transforms $(1,0)$ into $(2,5)$ and transforms $(0,1)$ into $(4,3)$?
   (b) (5%) What matrix transforms $(7,3)$ into $(1,0)$ and transforms $(5,6)$ into $(0,1)$?
   (c) (5%) Is matrix $\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$ similar to $\begin{bmatrix} 2 & 0 \\ 3 & 3 \end{bmatrix}$?
   (d) (5%) Is matrix $\begin{bmatrix} 4 & 5 \\ 2 & 4 \end{bmatrix}$ similar to $\begin{bmatrix} 3 & 2 \\ 9 & 4 \end{bmatrix}$?

2. (10%) In each part, compute the stated power of $A = \begin{bmatrix} 0.50 & 0.75 \\ 0.50 & 0.25 \end{bmatrix}$.
   (a) (5%) A^8.
   (b) (5%) A^∞.

3. (20%) Suppose that $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are vectors with the inner product of any two vectors $\langle \mathbf{u}, \mathbf{v} \rangle = 5$, $\langle \mathbf{v}, \mathbf{w} \rangle = -3$, $\langle \mathbf{w}, \mathbf{u} \rangle = 2$, and the vector length $\|\mathbf{u}\| = 7$, $\|\mathbf{v}\| = 2$, $\|\mathbf{w}\| = 1$. Evaluate each of the following expressions:
   (a) (5%) $\langle \mathbf{u} + \mathbf{v}, \mathbf{v} + \mathbf{w} \rangle$.
   (b) (5%) $\langle 3\mathbf{v} - \mathbf{w}, 2\mathbf{u} + 3\mathbf{w} \rangle$.
   (c) (5%) $\|3\mathbf{w} - \mathbf{v}\|$.
   (d) (5%) $\|\mathbf{u} - \mathbf{v} + 2\mathbf{w}\|$.

4. (15%) For each matrix A, find an invertible matrix S and a diagonal matrix D such that $S^{-1}AS = D$.
   (a) (7%) $A = \begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$.
   (b) (8%) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

5. (20%) Let $A = \begin{bmatrix} 2 & 1 & 5 \end{bmatrix}$.
   (a) (10%) Find the basis for the nullspace of A.
   (b) (5%) Find the basis for the column space of A.
   (c) (5%) Find the basis for the row space of A.

6. (15%) True or false.
   (a) (3%) Orthogonal vectors are always linearly independent.
   (b) (3%) An invertible matrix is always diagonalizable.
   (c) (3%) Two similar matrices share the same eigenvalues and eigenvectors.
   (d) (3%) A real symmetric matrix is always diagonalizable.
   (e) (3%) If A and B are orthogonal matrices, then $(BA)^T$ is also orthogonal.