科目：統計學 適用：經濟系

編號：312

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

1.簡單迴歸模型如右所示：$y_i = \beta + e_i$，$i = 1,2,3,...,n$. 回答以下問題。

(1)寫出該模型的假設條件。

(2)導出β的最小平方估計式$\hat{\beta}$。

(3)在(1)的假設條件下，證明$\hat{\beta}$是β的不偏估計式。

(每小題10分，共30分)

2.假設複迴歸模型為$y_i = \beta_1 + \beta_2 x_{i2} + \beta_3 x_{i3} + \beta_4 x_{i4} + e_i$，$i = 1,2,3,...,n$，且滿足所有迴歸的假設條件。

(1)說明以t統計量檢定$H_0 : \beta_2 + \beta_3 = 1$ vs. $H_1 : \beta_2 + \beta_3 \neq 1$的檢定程序。

(2)說明以F統計量檢定$H_0 : \beta_2 + \beta_3 = 1$ vs. $H_1 : \beta_2 + \beta_3 \neq 1$的檢定程序。(每小題10分，共20分)

3. (10%) A and B play the following game: A write down either number 1 or 2 and B must to guess which one. If the number that A has written down is i and B has guess correctly, B receives i dollars from A. If B makes a wrong guess, B pays1 dollar to A. If B randomizes his decision by guessing 1 with probability 1/2 and 2 with probability 1/2, determine his expected gain if (a) A has written down number 1 and (b) A has written down number 2.

4. (10%) You are expecting a friend to come at 8 o'clock, knowing that your friend will arrive at some time uniformly distributed between 8 and 8:30.

(a).**(5%)** What is the probability that you have to wait longer than 10 minutes?

(b).**(5%)** If at 10:10 your friend has not yet arrived, what is the probability that you will have to wait at least an additional 10 minutes?

5. (18%) The joint density function of X and Y is

$$f_{X,Y}(a,b)=\begin{cases} a+kb & if\ 0<a<1, 0<b<2 \\ 0 & otherwise \end{cases}$$

(a).**(6%)** What is the value of k such that $f_{X,Y}(a,b)$ is a probability density function?

(b).**(6%)** Are X and Y independent?

(c).**(6%)** Find $P\{X+Y<1\}$.

6. (12%) Mr. White proposed a method (White's test) to test whether a coin is unbiased as follows. Toss this coin 3 times independently, and conclude it is biased only if the head appear great or equal to 2 times. Please answer the following questions.

(a) .**(6%)** What is the type I error of White's test (assume the coin is unbiased)?

(b) .**(6%)** If the head appear with the probability 0.6 (the coin is indeed biased), what is the power of White's test?