科目：工程數學一（線性代數）

編號：343. 352　　適用：電機系、電機系(通訊工程)

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

(一) **True and False**; If **False**, give a "**reason**" or "**example**". (45 分, 每題 3 分)

(a) If A^2 is a symmetric matrix, then A is a symmetric matrix.

(b) An invertible matrix is always diagonalizable.

(c) n x n matrices A and B; If $|A - B| = 0$, then $|A| = |B|$.

(d) n x n matrices A and B; If $AB=O$, then $|A| = 0$ *or* $|B| = 0$.

(e) n x n matrices A; If A is nonsingular, it is row equivalent to the identity matrix.

(f) If invertible n x n matrices A and B; then $det(BAB^{-1})=det(B)$.

(g) If n x n matrices A and B; then $det(BA)=det(A)\cdot det(B)$.

(h) If n x n matrix A, r=constant; then $det(rA)=rdet(A)$.

(i) (1,2,0) and (0,1,2) are bases for $\boldsymbol{R}^3$.

(j) (1,2,2), (-1,2,1), and (0,8,6) are bases for $\boldsymbol{R}^3$.

(k) n x n matrix A, then Col A, Null A, and rank A are subspace sets.

(l) The rank of the matrix consists of (1,2,3), (0,1,2), and (2,5,8) is 3.

(m) m x n matrix A with rank r, **b** is a column vector; If n=r, then $A\boldsymbol{x}=\boldsymbol{b}$ has infinite many solutions.

(n) m x n matrices A and B, If rank A=rank B, then rank (A)=rank ($A+B$).

(o) $\{t, \sin(t), \cos(2t)\}$ is a linearly dependent set of function defined on $\boldsymbol{R}$.

(二) A linear transformation T: $\boldsymbol{R}^2 \rightarrow \boldsymbol{R}^3$ such that $T(1,1)=(1,0,1)$ and $T(2,3)=(1,-1,4)$ (20 分, 每題 5 分)

(a) Find $T(a,b)$ (Express using a and b)

(b) Find $T(c,c)$ (Express using c)

(c) Find $T(12,15)$ and $T(12,12)$

(d) If $T(k,l)=(2,-1,5)$, Find k and l.

(三) $A=\begin{bmatrix}1 & 0\\ 2 & -1\end{bmatrix}$ (35 分, 每題 5 分)

(a) Is matrix A Hermitian matrix or Normal matrix?

(b) Find det(A) and det(A^2).

(c) Find A^{-1} and A^T.

(d) Find the eigenvalues and corresponding eigenvectors of A.

(e) Find matrices P and D, so that $P^{-1}AP=D$, where D is a diagonal matrix.

(f) Find the minimum positive integer n, so that $A^n=I$, where I is an identity matrix.

(g) Find $A^{100}+A^{199}$.