1. (20 points)
Give a combinatorial proof for following equation. Assume $m, n \geq r$.

$$\sum_{0 \leq k \leq r} \binom{m}{k} \binom{n}{r-k} = \binom{m+n}{r}.$$

2. (10 points * 2)
Assume the universe consists of integers and let $p(x, y)$ denote $x|y$. Show and explain the truth values of the following terms.
(a) $\forall x \exists y\ p(x, y)$
(b) $\exists x \forall y\ p(x, y)$
Note: $x|y$ means $\exists z\ y = xz$.

3. (20 points)
Solve the recurrence

$$Q_0 = \alpha;\ \ Q_1 = \beta;$$
$$Q_n = (1 + Q_{n-1})/Q_{n-2},\ \text{for } n > 1.$$

Assume that $Q_n \neq 0$ for all $n \geq 0$.

4. (10 points * 2)
Assume $A = \{a, b, c, d, e, f, g\}$, $B = \{u, v, x, y, z\}$.
(a) Construct an onto function $f : A \to B$.
(b) For the function f defined above, the pre-images of individual elements of B make a partition of A. Construct a bijective function which is from such partition of A to B.
Note: a pre-image of a subset of B is $\{x \in A | f(x) \in B\}$.

5. (20 points)
Given two partially ordered set, $(A, \mathscr{R}_1)$ and $(B, \mathscr{R}_2)$, a relation $\mathscr{R}$ on $A \times B$ is defined as: $(a, b)\mathscr{R}(x, y)$ if $a\mathscr{R}_1 x$ and $b\mathscr{R}_2 y$.
Prove that $\mathscr{R}$ is a partial order.