國立暨南國際大學 104 學年度轉學生入學考試試題

第 2 節

科目：微積分 適用：土木系二. 電機系二. 應化系二. 應光系二

編號：311、331、342、351

考生注意：
1.依次序作答，只要標明題號，不必抄題。
2.答案必須寫在答案卷上，否則不予計分。
3.限用藍、黑色筆作答；試題須隨卷繳回。

本試題共 2 頁 第 1 頁

(以下各題均須寫出計算過程方予計分)

1. (10%) Let f be continuous on $[a, b]$. A function G is called an *antiderivative* for f on $[a, b]$ if and only if: G is continuous on $[a, b]$ and $G'(x) = f(x)$ for all $x \in (a, b)$.
   (a) (5%) Describe the fundamental theorem of integral calculus.
   (b) (5%) Use part (a) to evaluate $\int_1^4 x^2 dx$.

2. (10%) Assume that f is a continuous function and that $\int_0^x f(t)dt = \frac{2x}{4+x^2}$. Find the function $f(x)$ and sketch the graph of it.

3. (a) (5%) Evaluate $\int_0^{\sqrt{3}} x^5\sqrt{x^2+1}dx$. (b) (5%) Show that $\int_0^1 xe^x dx = 1$.

4. (10%) Find the volume of the solid generated by revolving the region between $y=x^2$ and $y=2x$ about the y-axis.

5. (10%) Derive the formula $\int_0^{\infty} \frac{1}{\sqrt{\pi N}} e^{-\frac{(x+\sqrt{E})^2}{N}} dx = \int_{\sqrt{2E/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$.

6. (10%) Sketch the curve $r = 1-\cos\theta$ given in polar coordinates, and find the area enclosed by the curve.

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7. (10%) Show that

   (a) (5%) $\lim_{x\to 0+} \sqrt{x}\ln x = 0$

   (b) (5%) $\lim_{n\to\infty} (3^n + 4^n)^{1/n} = 4$

8. (10%) Let $f(x) = xe^x$.

   (a) (5%) Find a power series representation of f in powers of x.

   (b) (5%) Integrate the power series in part (a) and show that

   $$\sum_{n=1}^{\infty} \frac{1}{n!(n+2)} = \frac{1}{2}.$$

9. (10%) Find the directional derivative of $f(x, y) = (x-1)y^2 e^{xy}$ at (0, 1) toward the point (-1, 3).

10. (10%) Determine the path of steepest descent along the surface $z = x^2+3y^2$ from the point (1, -2, 13).